Quantitative Conclusions from Heavy-Ion Collisions

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MADAI Collaboration
Models and Data Analysis Initiative
http://madai.us

Cyber-enabled Discovery and Innovation

MICHIGAN STATE UNIVERSITY Duke UNIVERSITY THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL renci

1st MADAI Collaboration Meeting, SANDIA 2010

Goals

1. Determine fundamental properties (parameters) describing super-hadronic matter from experimental data

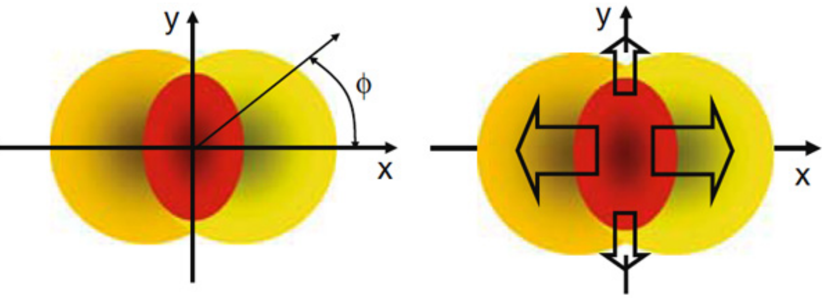
OR

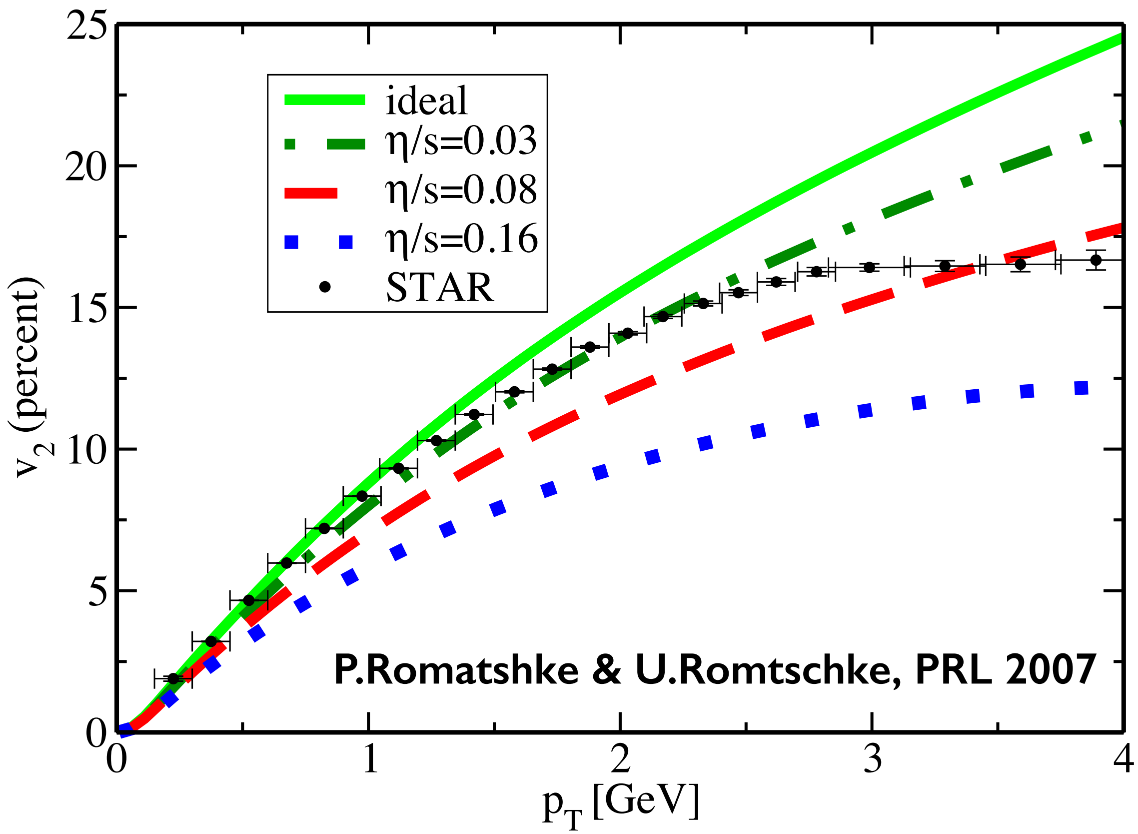
2. Validate that matter in H.I. collisions behaves consistently with known properties of equilibrated system (from lattice)

How this was done before ($v2$ and η/s)

Study single parameter vs. single observable

$$v_2 \equiv \langle \cos 2\phi \rangle$$





PROBLEM

v2 depends on ....

- viscosity
- saturation model
- pre-thermal flow
- Eq. of State
- T-dependence of η/s
- initial T_{xx}/T_{zz}
- . . . .

e.g. Drescher, Dumitru, Gombeaud and Ollitrault
PRC 2007

Correct Way (MCMC)

- Simultaneously vary *N* model parameters $\boldsymbol{x_i}$
- Perform random walk weight by likelihood

$$\mathcal{L}(\mathbf{x}|\mathbf{y}) \sim \exp\left\{-\sum_a \frac{(y_a^{(\mathrm{model})}(\mathbf{x}) - y_a^{(\mathrm{exp})})^2}{2\sigma_a^2}\right\}$$

- Use all observables $\boldsymbol{y_a}$
- Obtain representative sample of posterior

Very Difficult Because…

I. Too Many Model Runs
Requires running model ~10^6 times

II. Many Observables
Could be hundreds of plots,
each with dozens of points
Complicated Error Matrices

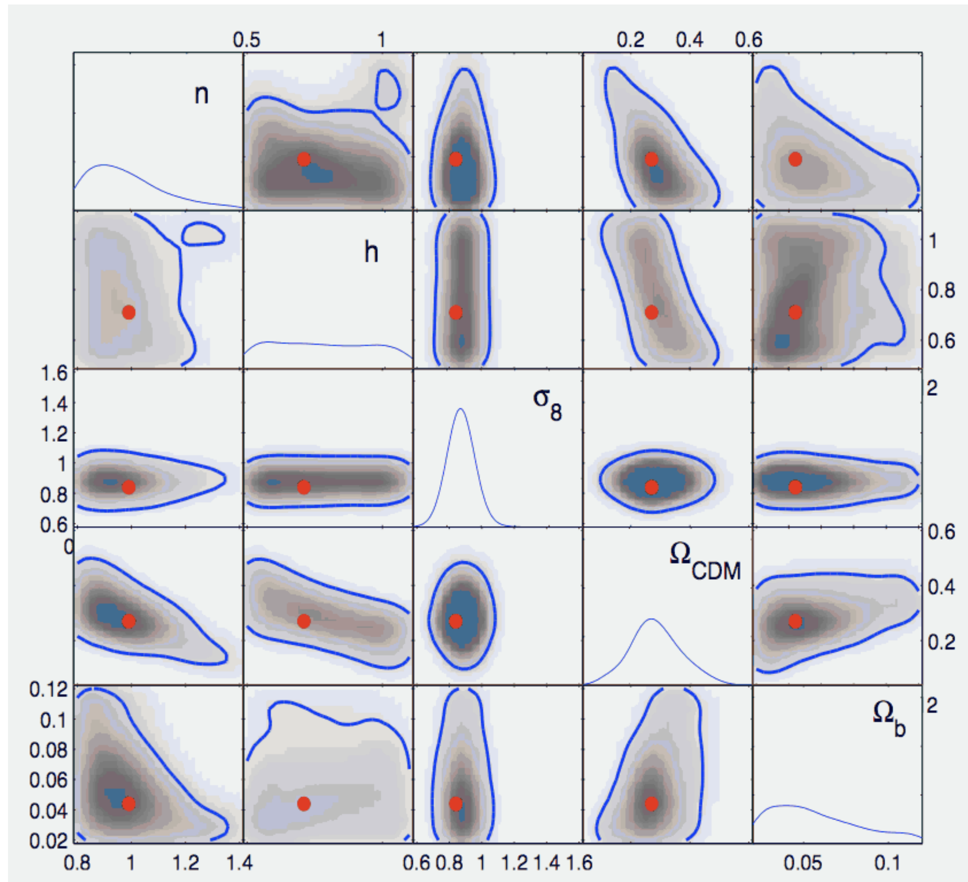
Model Emulators

1. **Run the model ~1000 times**
   Semi-random points (LHS sampling)

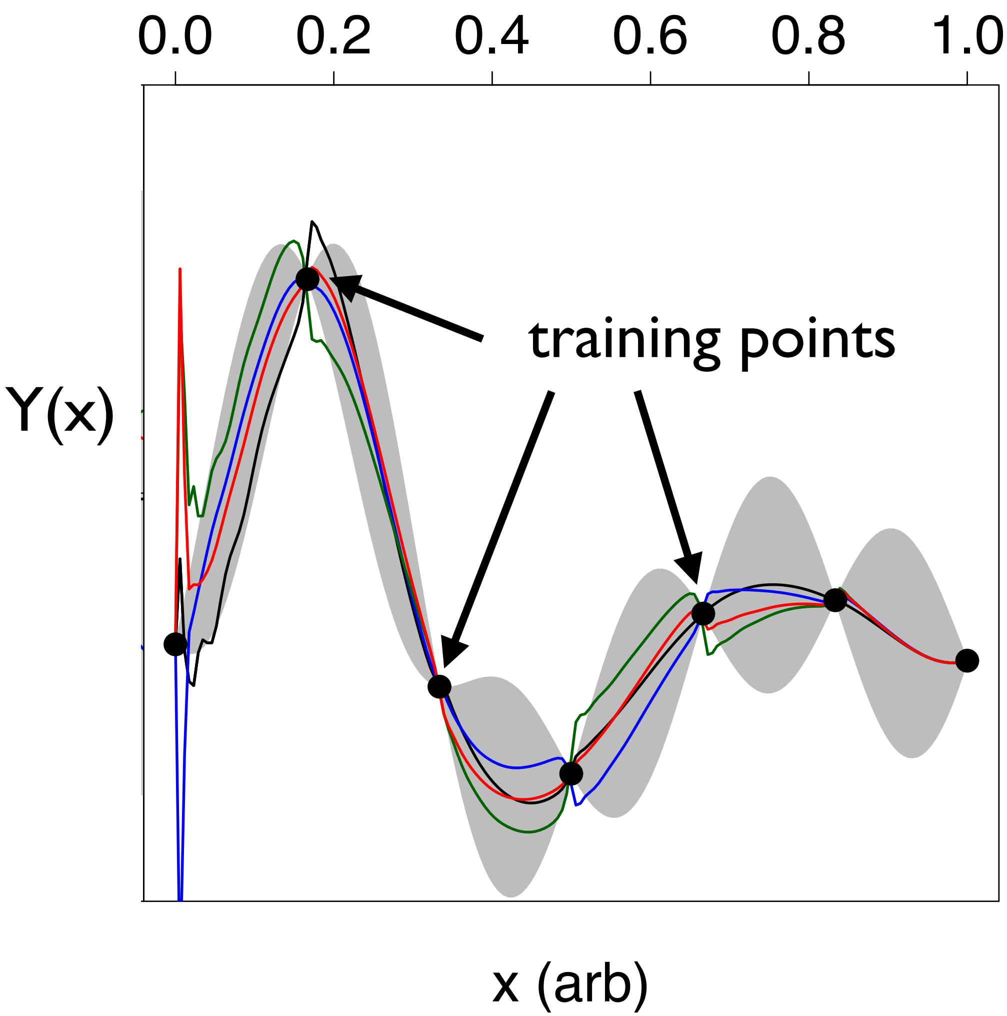
2. **Determine Principal Components**
   $(y_a - \langle y_a \rangle)/\sigma_a \rightarrow z_a$

3. **Emulate z_a (Interpolate) for MCMC**
   Gaussian Process…

$$\mathcal{L}(\mathbf{x}|\mathbf{y}) \sim \exp\left\{-\frac{1}{2}\sum_a (z_a^{(\mathrm{emulator})}(\mathbf{x}) - z_a^{(\exp)})^2\right\}$$



S. Habib,K.Heitman,D.Higdon,C.Nakhleh&B.Williams, PRD(2007)



Emulator Algorithms

- Gaussian Process
  - Reproduces training points
  - Assumes localized Gaussian covariance
  - Must be trained, i.e. find "hyper parameters"
- Other methods also work

14 Parameters

- 5 for Initial Conditions at RHIC
- 5 for Initial Conditions at LHC
- 2 for Viscosity
- 2 for Eq. of State

30 Observables

- π, K, p Spectra
  $\langle p_t \rangle$, Yields
- Interferometric Source Sizes
- v_2 Weighted by p_t

Initial State Parameters

$$\epsilon(\tau = 0.8\text{fm}/c) = f_{\text{wn}}\epsilon_{\text{wn}} + (1 - f_{\text{wn}})\epsilon_{\text{cgc}},$$

$$\epsilon_{\text{wn}} = \epsilon_0 T_A \frac{\sigma_{\text{nn}}}{2\sigma_{\text{sat}}} \left\{1 - \exp\left(-\sigma_{\text{sat}} T_B\right)\right\} + (A \leftrightarrow B)$$

$$\epsilon_{\text{cgc}} = \epsilon_0 T_{\text{min}} \frac{\sigma_{\text{nn}}}{\sigma_{\text{sat}}} \left\{1 - \exp\left(-\sigma_{\text{sat}} T_{\text{max}}\right)\right\}$$

$$T_{\text{min}} \equiv \frac{T_A T_B}{T_A + T_B},$$

$$T_{\text{max}} \equiv T_A + T_B,$$

$$u_\perp = \alpha\tau \frac{\partial T_{00}}{2T_{00}}$$

$$T_{zz} = \gamma P$$

5 parameters for RHIC, 5 for LHC

Equation of State and Viscosity

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

$$X_0 = X' R c_s(\epsilon)\sqrt{12},$$

$$x \equiv \ln \epsilon/\epsilon_h$$

$$\frac{\eta}{s} = \left.\frac{\eta}{s}\right|_{T=165} + \kappa \ln(T/165)$$

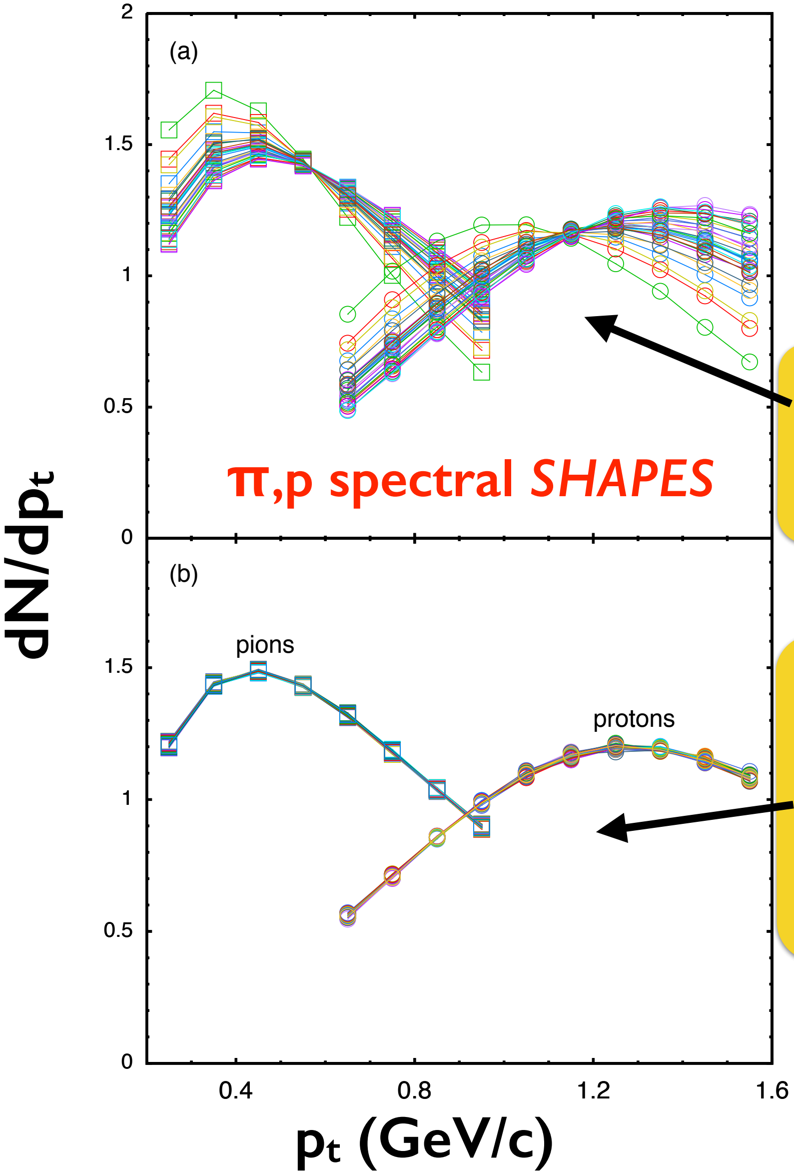
2 parameters for EoS, 2 for η/s

DATA Distillation

1. Experiments reduce PBs to 100s of plots
2. Choose which data to analyze
   Does physics *factorize*?
3. Reduce plots to a few representative numbers, y_a
4. Transform to principal components

Checking the Distillation

Spectral information encapsulated by two numbers, dN/dy & $\langle p_t \rangle$



model spectra from 30 random points in parameter prior

74 pion spectra: with $573 < \langle p_t \rangle_\pi < 575$ MeV

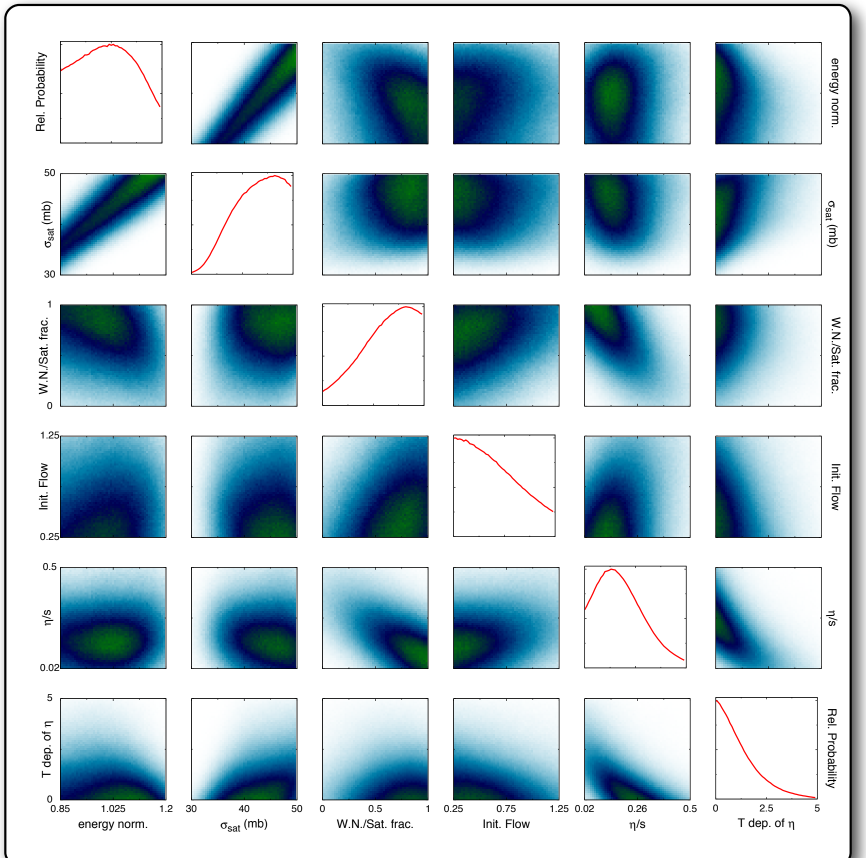
44 proton spectra: with $1150 < \langle p_t \rangle_p < 1152$ MeV

Two Calculations

J.Novak, K. Novak, S.P., C.Coleman-Smith & R.Wolpert, PRC 2014

RHIC Au+Au Data

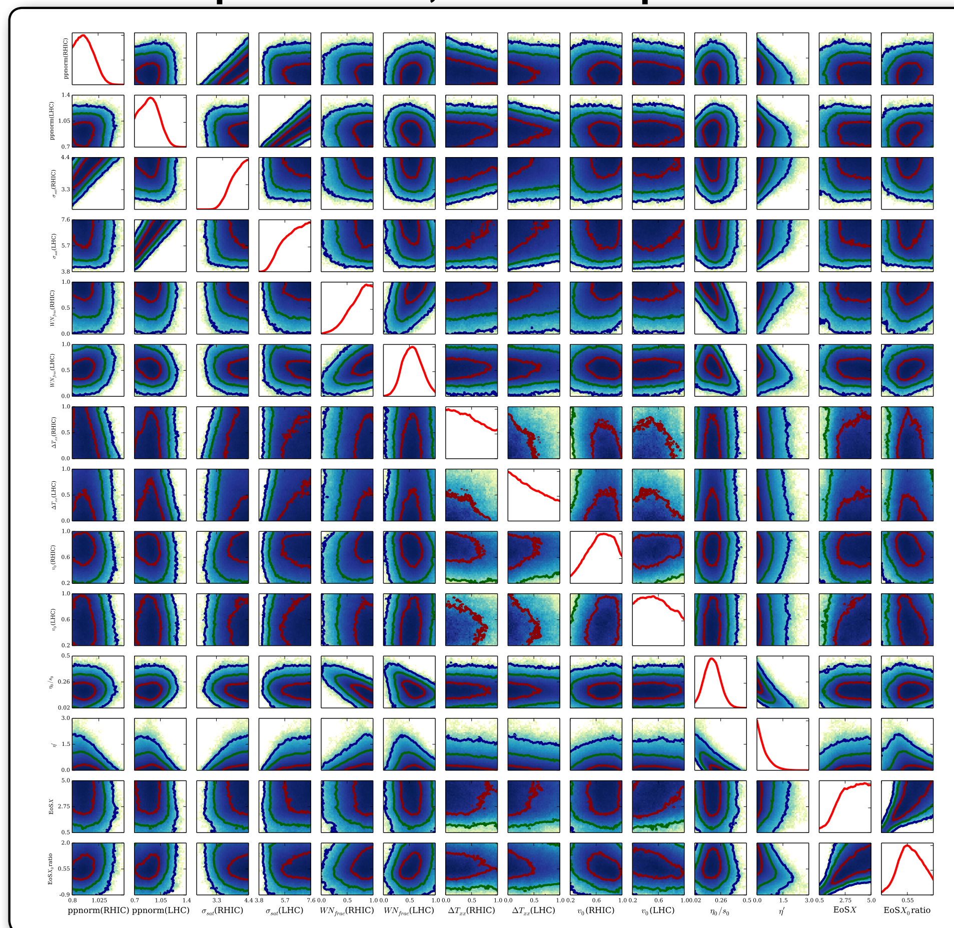
6 parameters



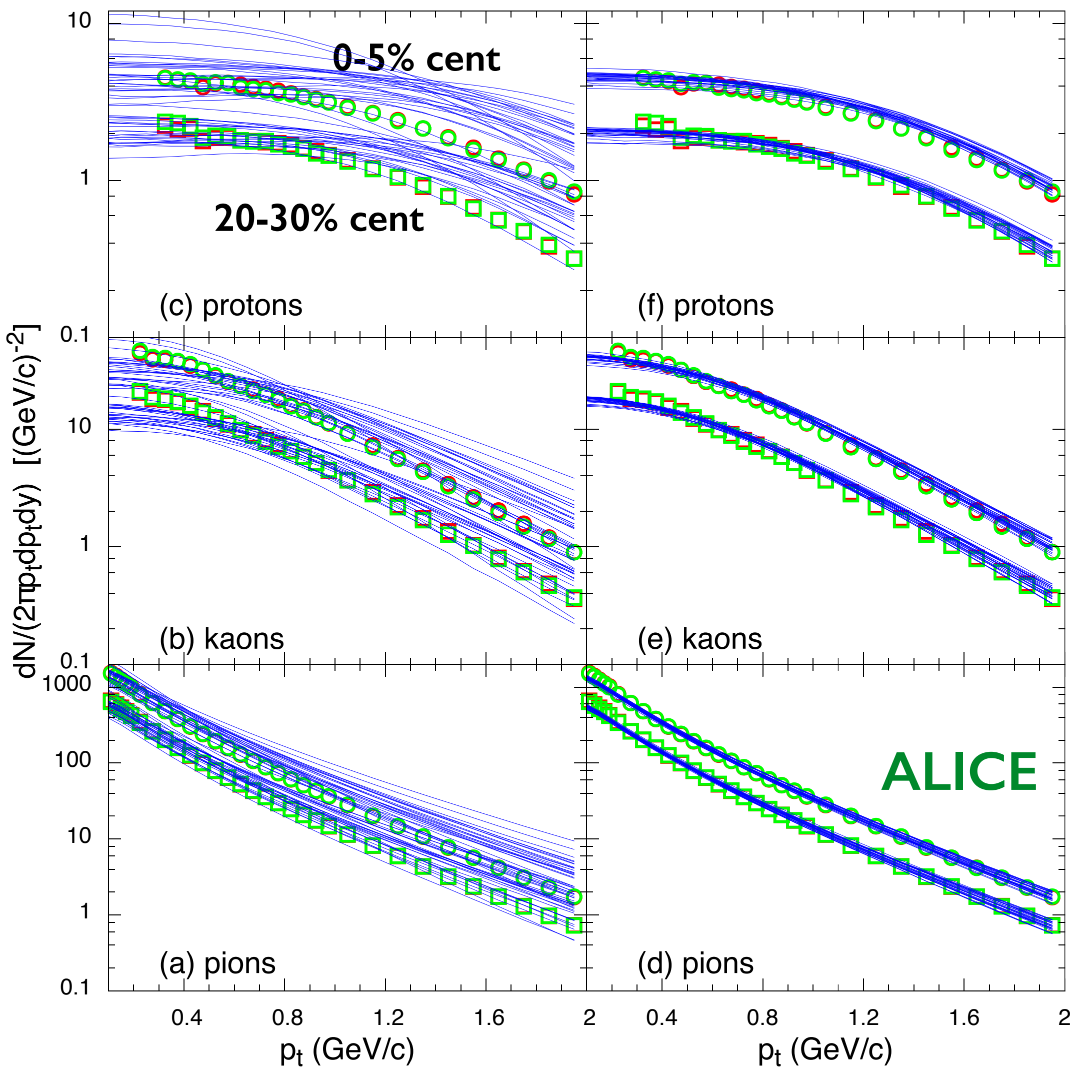
S.P., E.Sangaline, P.Sorensen & H.Wang, PRL 2015

RHIC Au+Au and LHC Pb+Pb Data

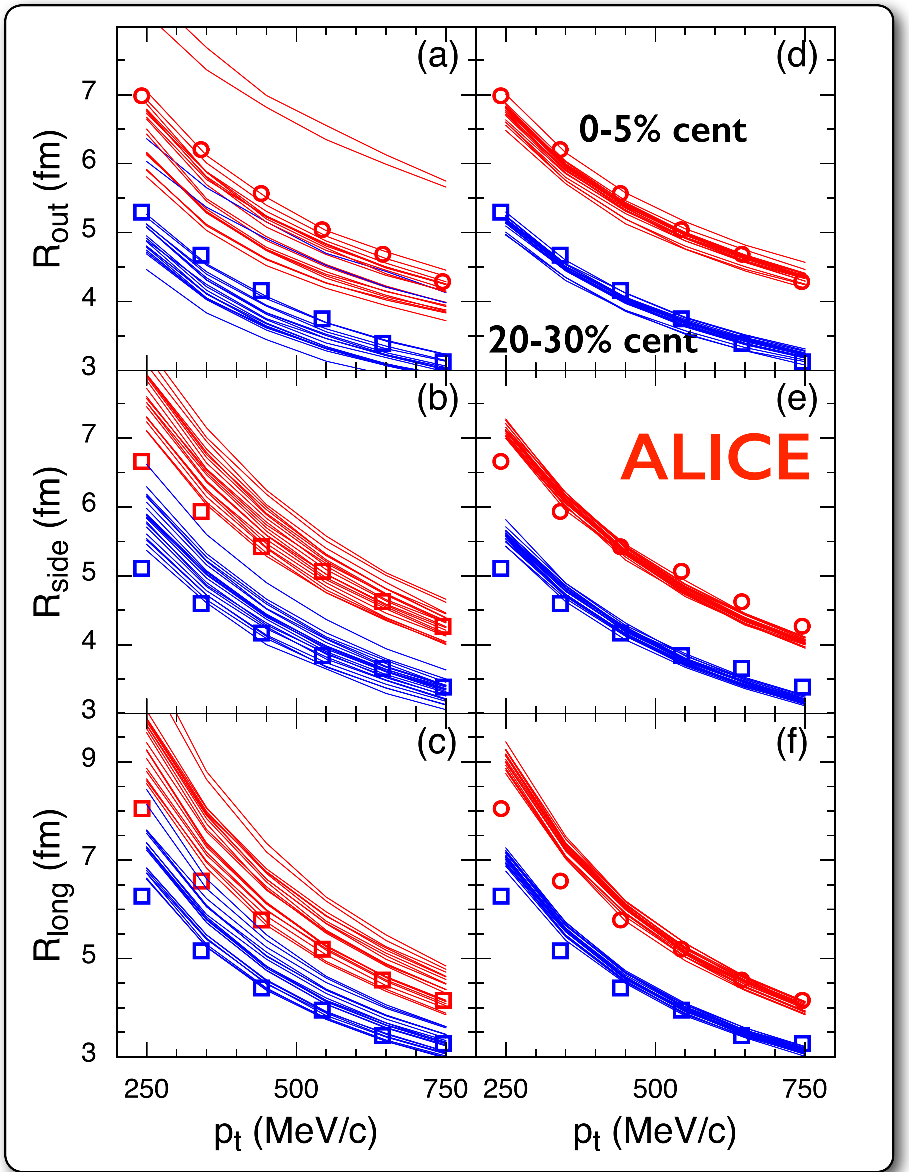
14 parameters, include Eq. of State



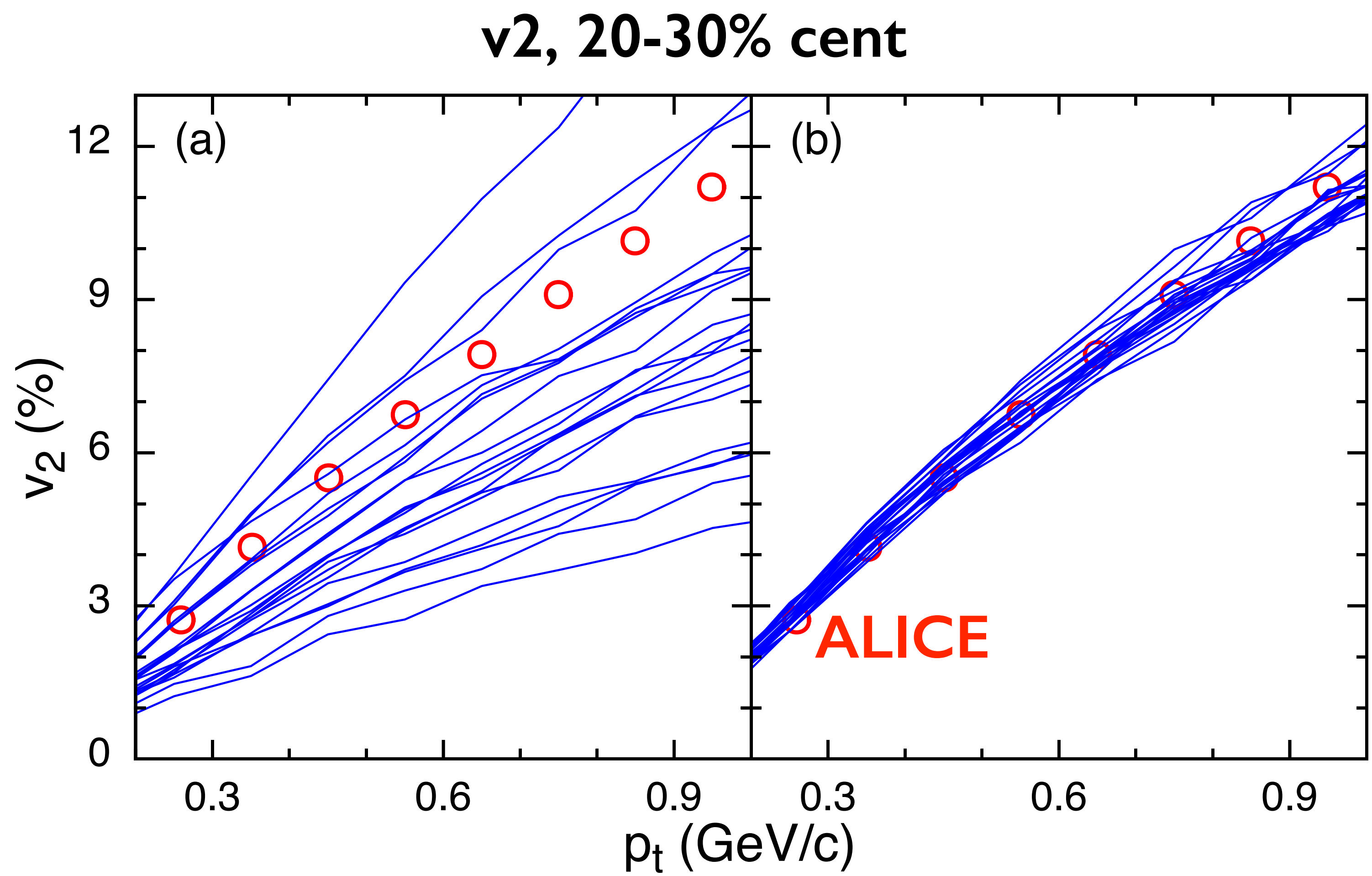
Sample Spectra from Prior and Posterior



Sample HBT from Prior and Posterior



Sample V2 from Prior and Posterior



Eq. of State

$$c_s^2(\epsilon) = c_s^2(\epsilon_h) + \left(\frac{1}{3} - c_s^2(\epsilon_h)\right) \frac{X_0 x + x^2}{X_0 x + x^2 + X'^2},$$

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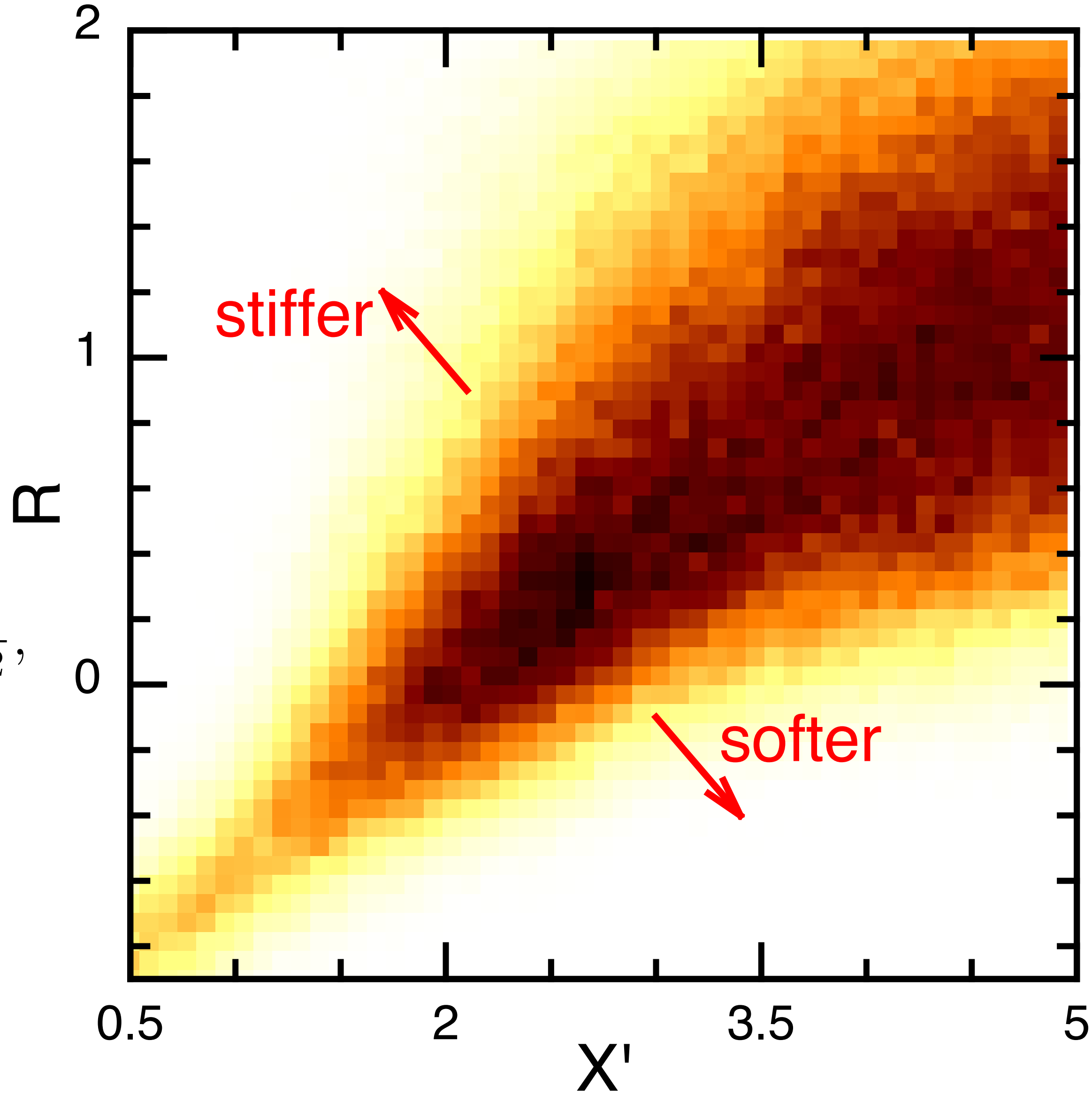
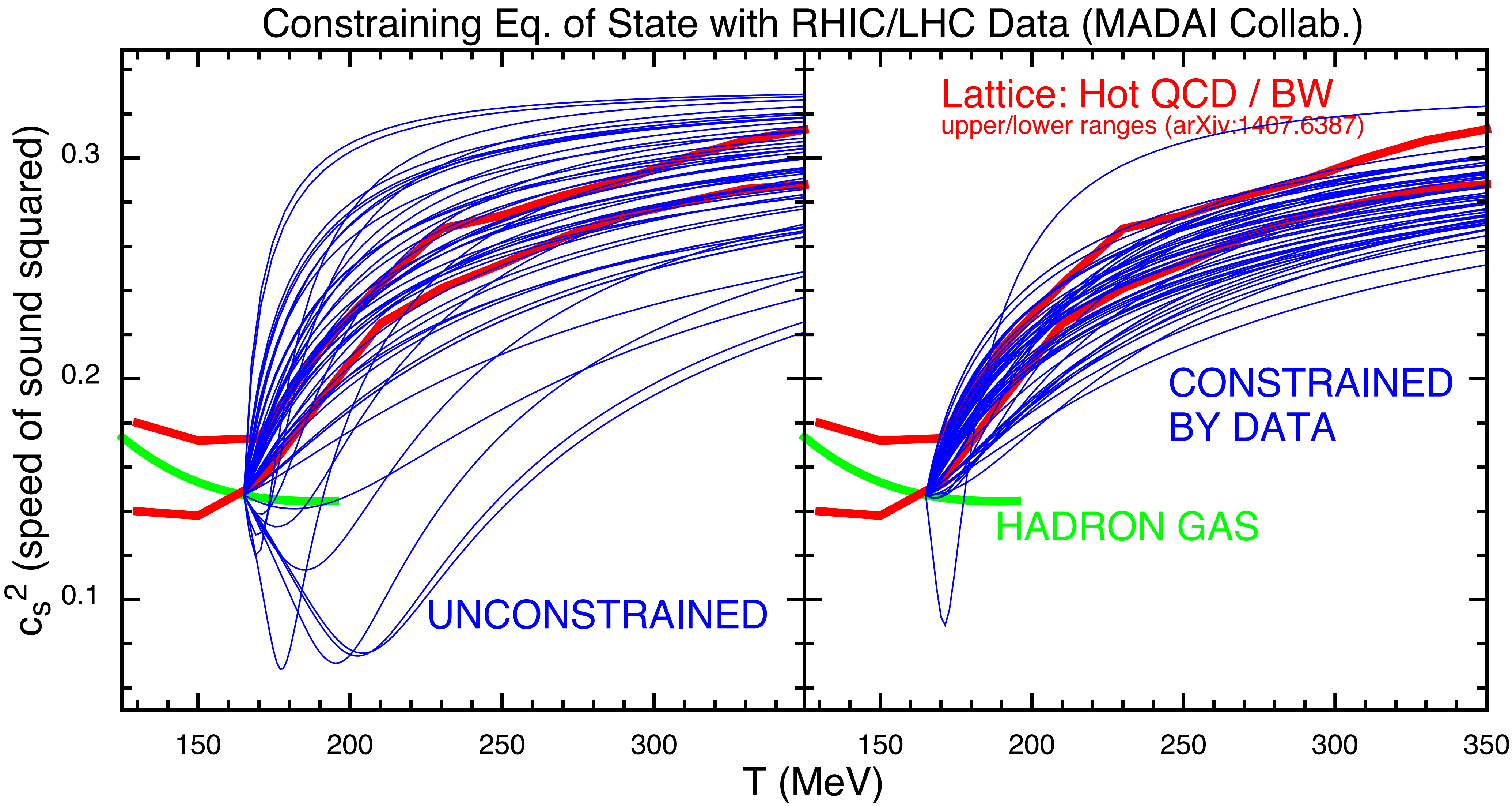
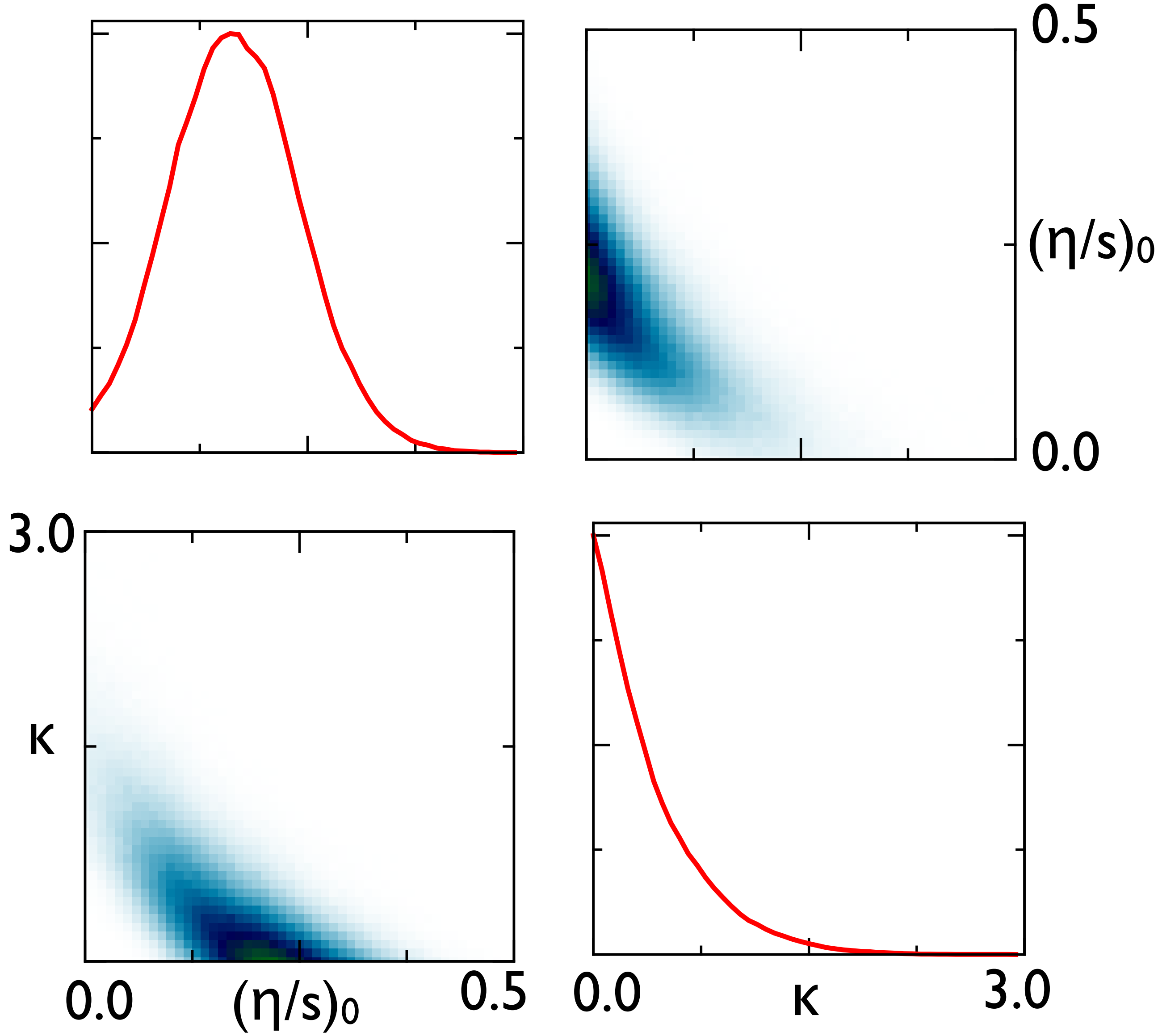


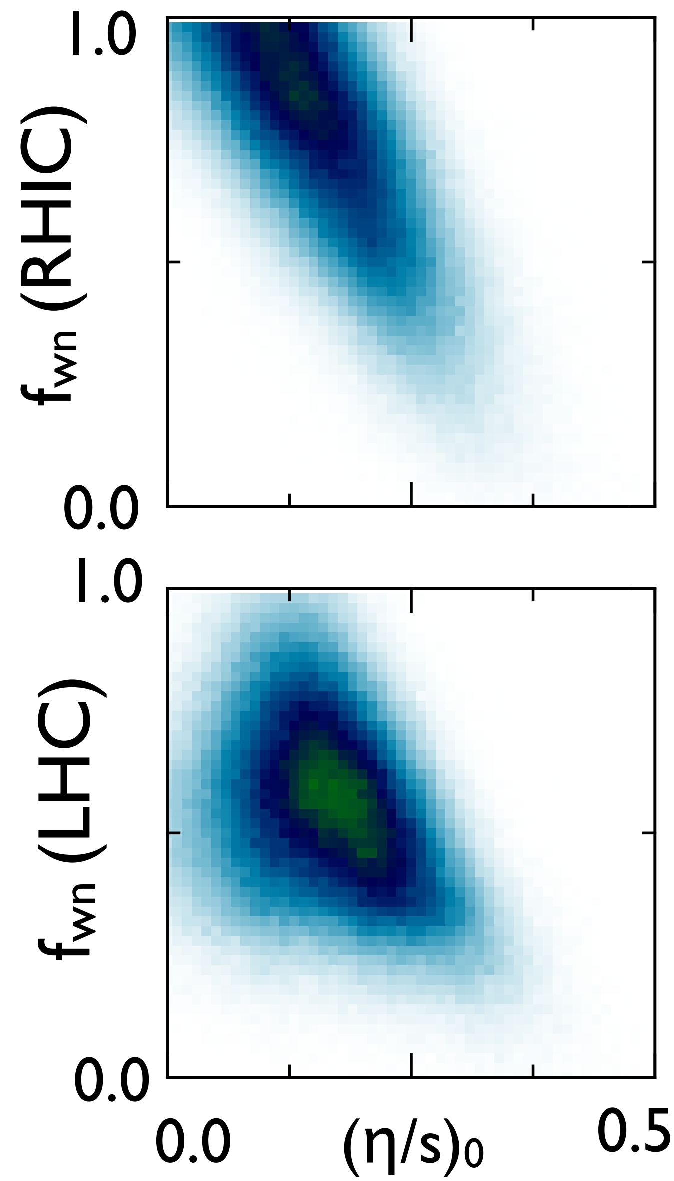

Constraining Eq. of State with RHIC/LHC Data (MADAI Collab.)
Lattice: Hot QCD / BW
upper/lower ranges (arXiv:1407.6387)
UNCONSTRAINED
CONSTRAINED
BY DATA
HADRON GAS
c_s^2 (speed of sound squared)
0.1
0.2
0.3
150
200
250
300
350
T (MeV)

η/s(T)

$$\eta/s = (\eta/s)_0 + \kappa \ln(T/165)$$



η/s vs saturation picture

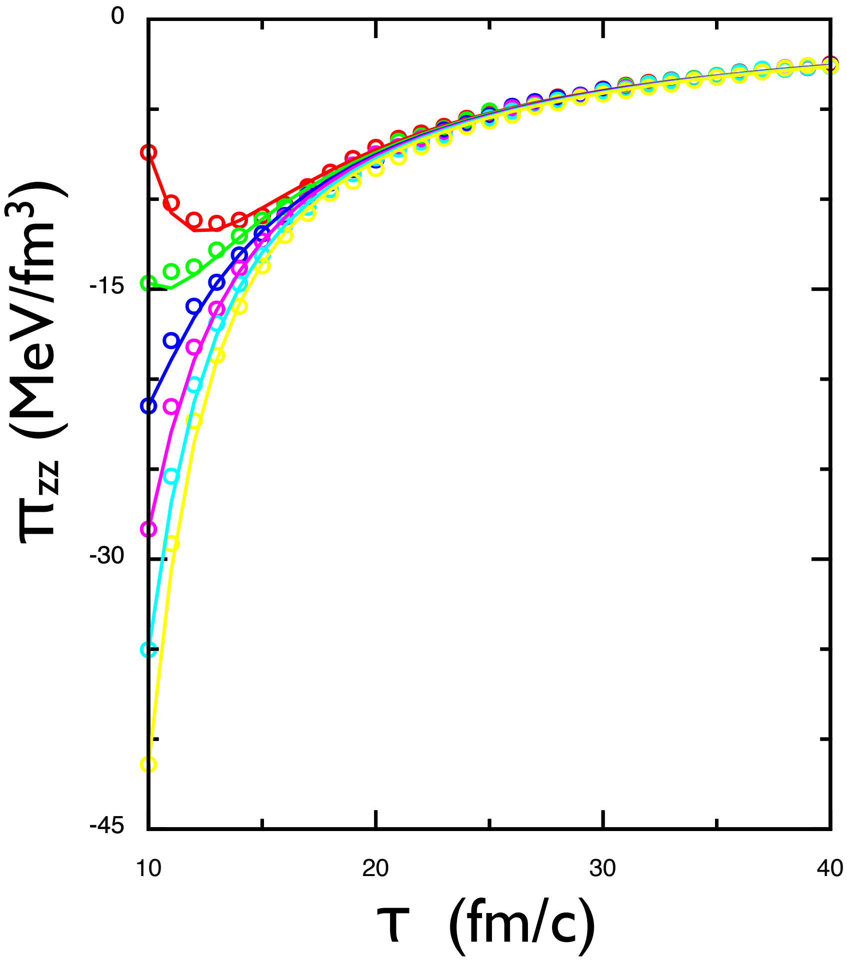


See Drescher, Dumitru, Gombeaud and Ollitrault
PRC 2007

What should you expect for η/s at T=165 MeV?

- ADS/CFT: 0.08
- Perturbative QCD: > 0.5 ($\sigma \approx$ 3 mb)
- Hadron Gas: $\approx$ 0.2 ($\sigma \approx$ 30 mb)

Viscosity from Hadron Cascade P.Romatschke & S.P. (ArXiv



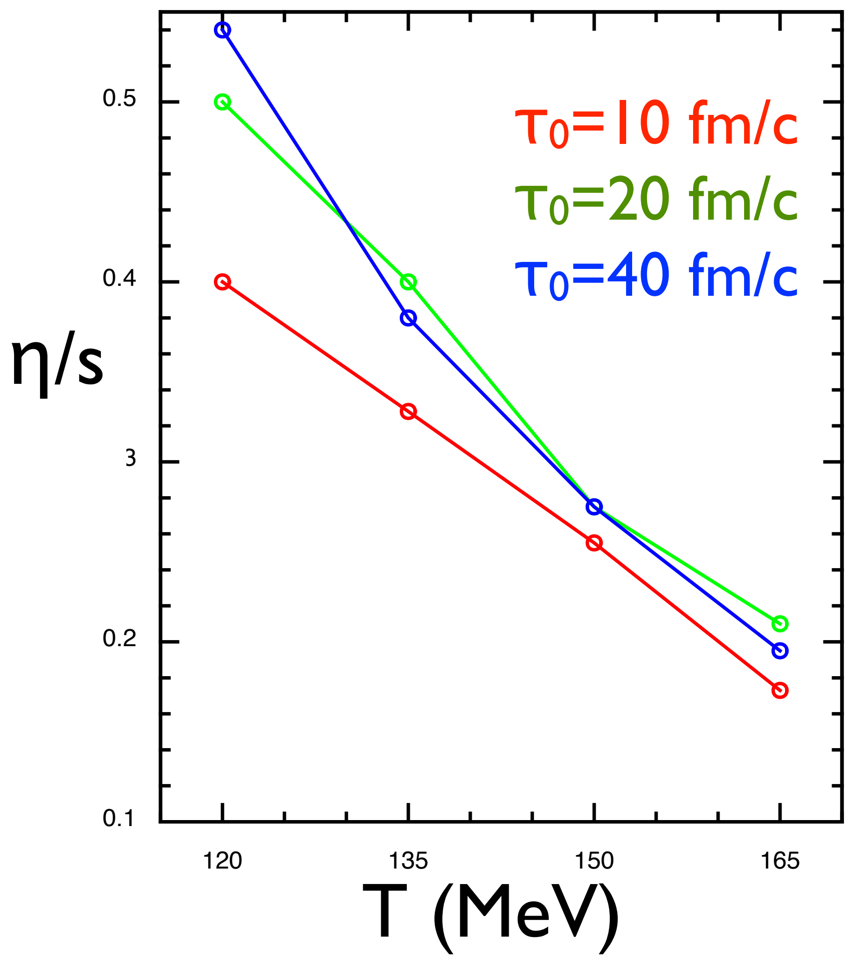
π_{zz} from B3D vs. Israel Stewart

$$\frac{d}{d\tau}\frac{\pi_{zz}}{\sigma_P} = -\frac{1}{\tau_{IS}}\frac{(\pi_{zz}-\pi_{zz}^{(NS)})}{\sigma_P},$$

$$\sigma_P^2(T) \equiv s\sum_i (2S_i+1)\int \frac{d^3p}{(2\pi)^3} f_i(p)\frac{p_x^2 p_y^2}{E_p}$$

$$\tau_{IS}(T) = \frac{\eta s T}{\sigma_P^2}$$

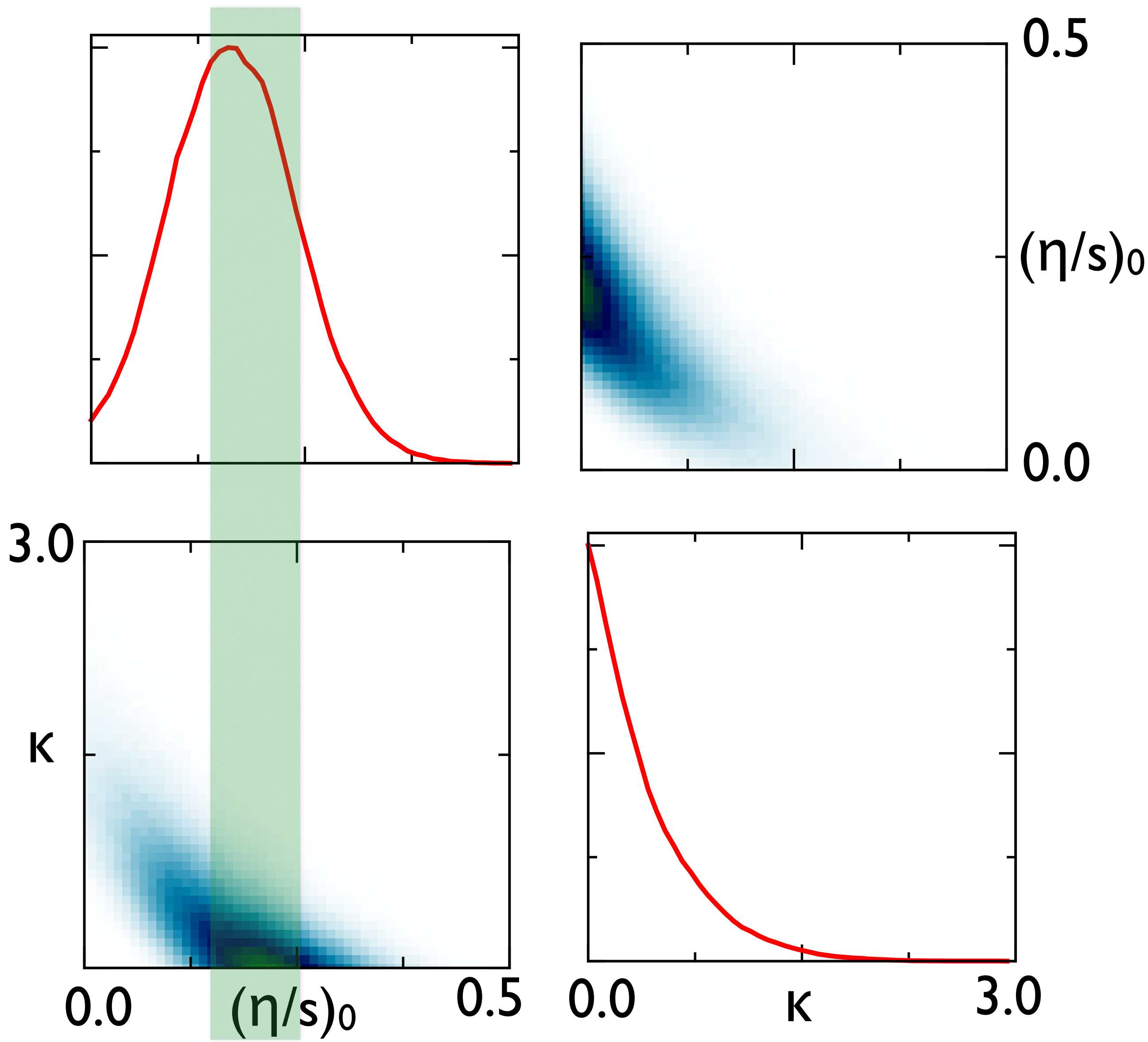
Viscosity from Hadron Cascade P.Romatschke & S.P. (ArXiv



- Sensitive to cross sections
- Consistent with $\tau_{IS} \sim 1.9\, \tau_{coll}$

For hadron gas ,
$0.15 < \eta/s\ (T=165) < 0.25$

Extracted η/s at T=165 MeV consistent with expectations for hadron gas!



CONCLUSIONS

- Robust
- Emulation works splendidly
- Scales well to more parameters & more data
- Eq. of State and Viscosity can be extracted from RHIC & LHC data
- Other parameters not as well constrained
- Heavy-Ion Physics can be a Quantitative Science!!!!

FUTURE

- Improve statement of uncertainties
- Add parameters
  many related to hadronization region
- Consider more data
  - more observables
  - data from different beams/energies
- Improve models
  - Lumpy initial conditions
  - 3D calculations for lower energies
  - Fill in missing physics (especially near hadronization)

If you're interested…

1. Tools are easily extended
2. Download software and tutorial from http://madai.us
3. Talk to me (prattsc@msu.edu)
   or Evan Sangaline (esangaline@gmail.com)

Made possible by contributions from DOE, NSF, Chris Coleman-Smith, John Novak, Kevin Novak, Evan Sangaline, Paul Sorensen, Joshua Vredevoogd , Hui Wang, Robert Wolpert, and viewers like you.

Additional slide: Charge BFs and charge susceptibilities

S.P., C. Ratti and W.McCormack, arXiv:1409.2164