Hadron Physics and QCD (theory)

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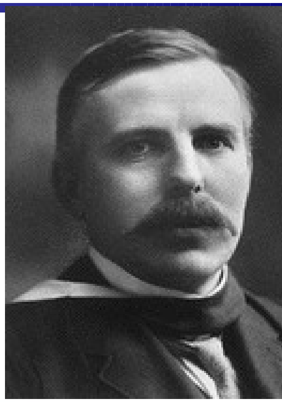
- References for this lecture
  - G. Sterman, *Partons, Factorization and Resummation*, hep-ph/9606312
  - John Collins, *The Foundations of Perturbative QCD*, published by Cambridge, 2011
  - CTEQ, *Handbook of perturbative QCD*, Rev. Mod. Phys. 67, 157 (1995).
- General references
  - CTEQ web site: http://www.phys.psu.edu/~cteq/

Outline

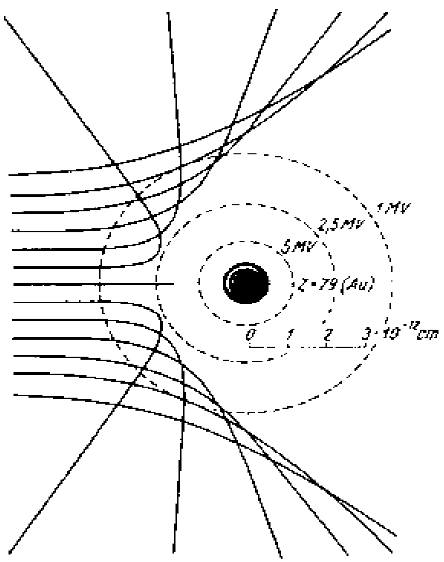
- I. General Introduction: Brief History and Basics of Basics
- II. Hadronic processes to study nucleon structure
- IV. Factorization and QCD dynamics

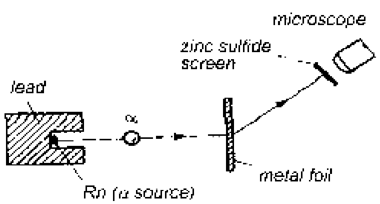
Rutherford scattering

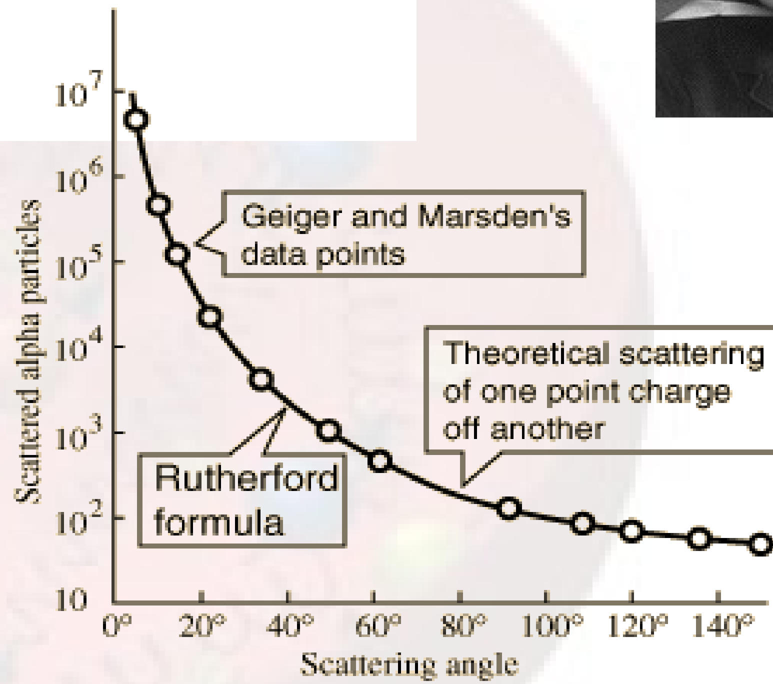


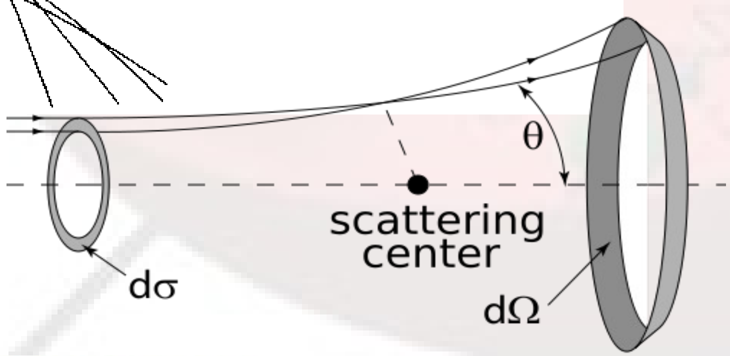
The Scattering of α and β Particles by Matter and the Structure of the Atom

E. Rutherford, F.R.S.*
Philosophical Magazine
Series 6, vol. 21
May 1911, p. 669-688



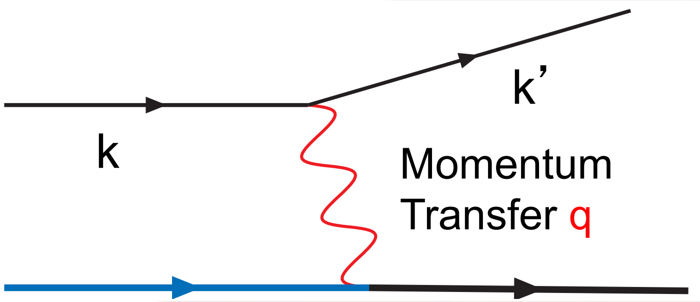






$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha\hbar c}{2mv_0^2}\right)^2 \frac{1}{\sin^4(\theta/2)}$$

Power counting analysis



$$2E_{k'}\frac{d\sigma}{d^3k'} \propto |\mathcal{M}|^2 \quad \mathcal{M} \propto \frac{1}{q^2}$$

$$q^2 = -Q^2 \approx E_k E'_k \sin^2\frac{\theta}{2}$$

- EM interaction perturbation, leading order dominance, potential~1/r
- Point-like structure
- Powerful tool to study inner structure

Basic idea of nuclear science

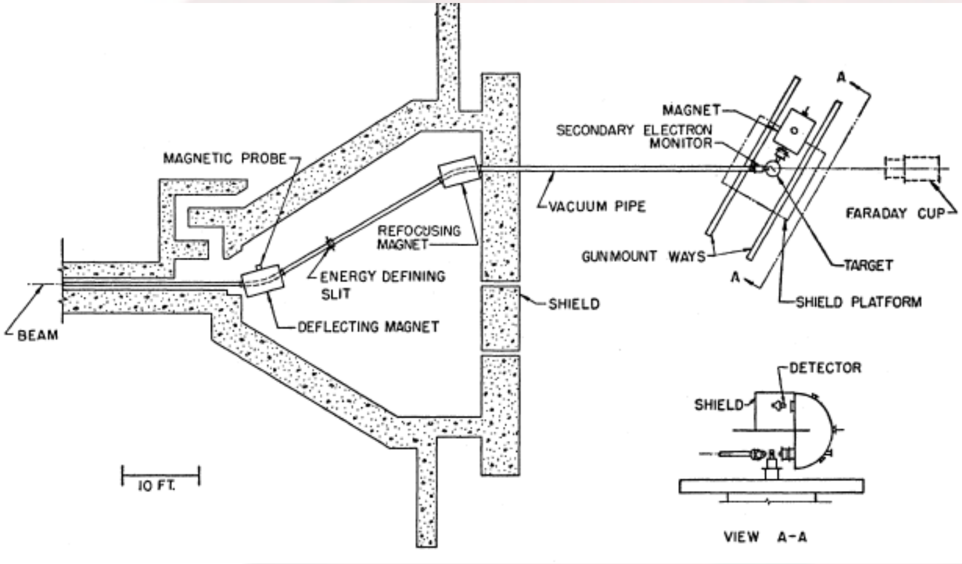
Since the α and β particles traverse the atom, it should be possible from a close study of the nature of the deflexion to form some idea of the constitution of the atom to produce the effects observed. In fact, the scattering of high-speed charged particles by the atoms of matter is one of the most promising methods of attack of this problem. The develop-

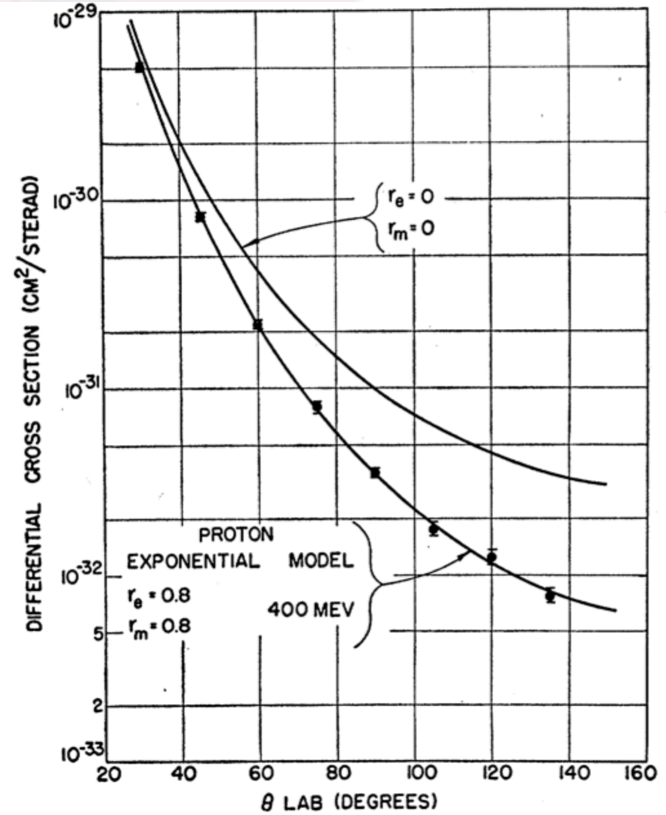
Rutherford, 1911

Finite size of nucleon (charge radius)

Hofstadter

- Rutherford scattering with electron

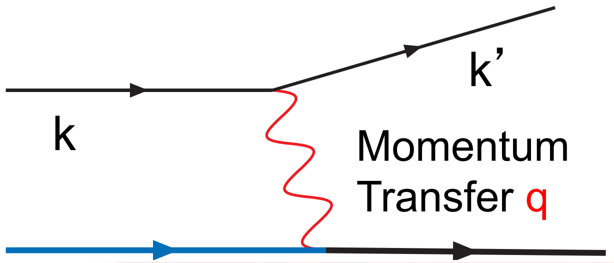




Renewed interest on proton radius:
μ-Atom vs e-Atom (EM-form factor)

RevModPhys.28.214

Current-current interactions

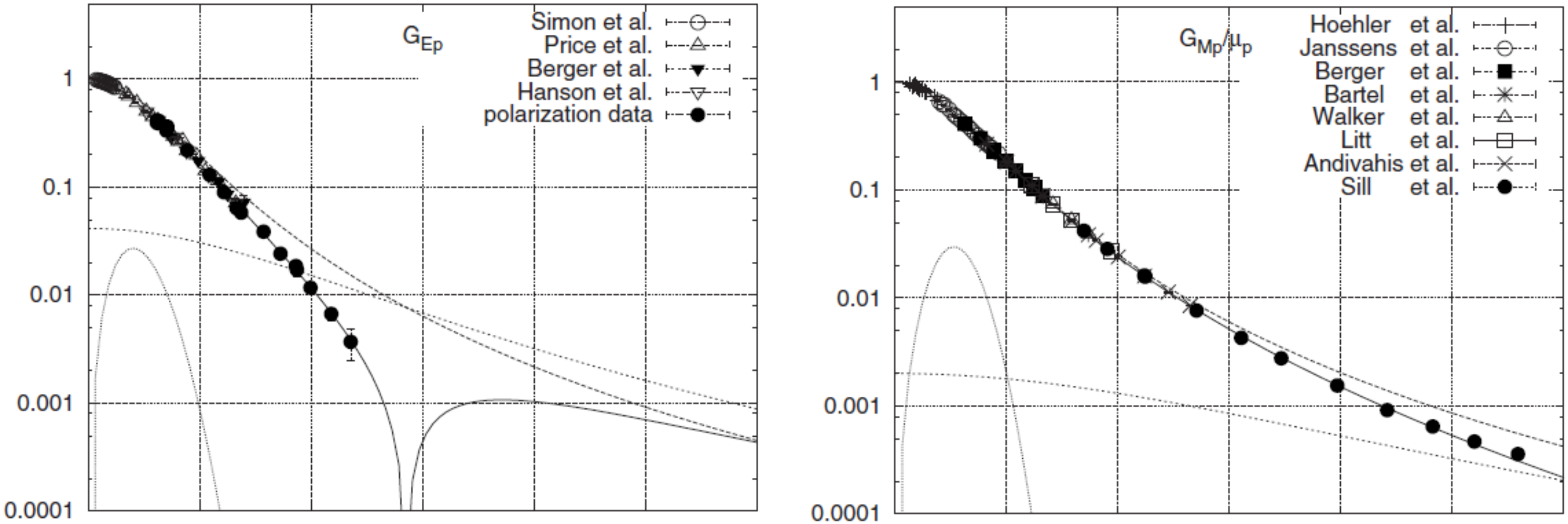


$$J^{\ell}_{\mu} = \bar{\psi}(k')\gamma_\mu \psi(k)$$

$$J^{h}_{\mu} = \bar{U}(P')\left[\gamma_\mu F_1(Q^2) + i(\sigma_{\mu\nu} q^\nu / 2M) F_2(Q^2)\right] U(P)$$

- The deviation is characterized as the nucleon form factors
- Power behavior for the Dirac Form Factor, $F_1 \sim 1/Q^4$, Pauli Form Factor F_2 is further suppressed at large Q^2

More data on elastic scattering



$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2),$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2),$$

$$\tau = Q^2/4M^2$$

Quark model


Gell-Man

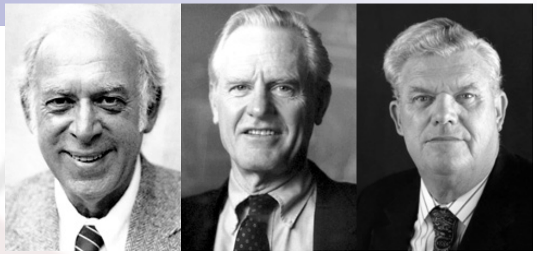
- Nucleons, and other hadrons are not fundamental particles, they have constituents
- Gell-Man Quark Model
  - Quark: spin ½
    - Charges: up (2/3), down (-1/3), strange (-1/3)
  - Flavor symmetry to classify the hadrons
    - Mesons: quark-antiquark
    - Baryons: three-quark
    - Gell-Man-Okubo Formula

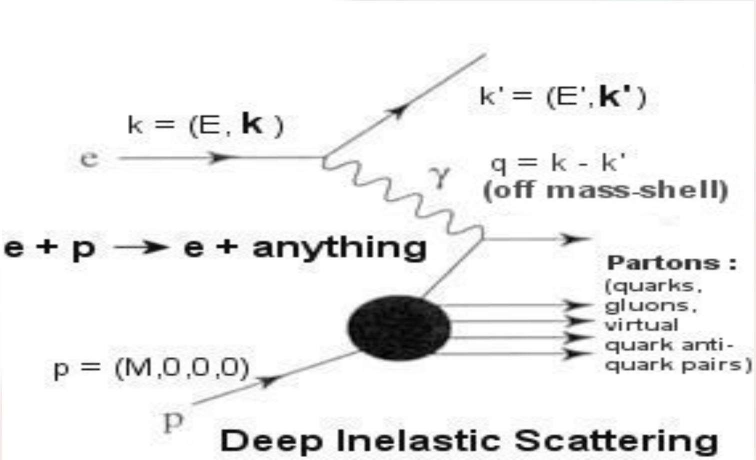
Nucleons are not fundamental particles

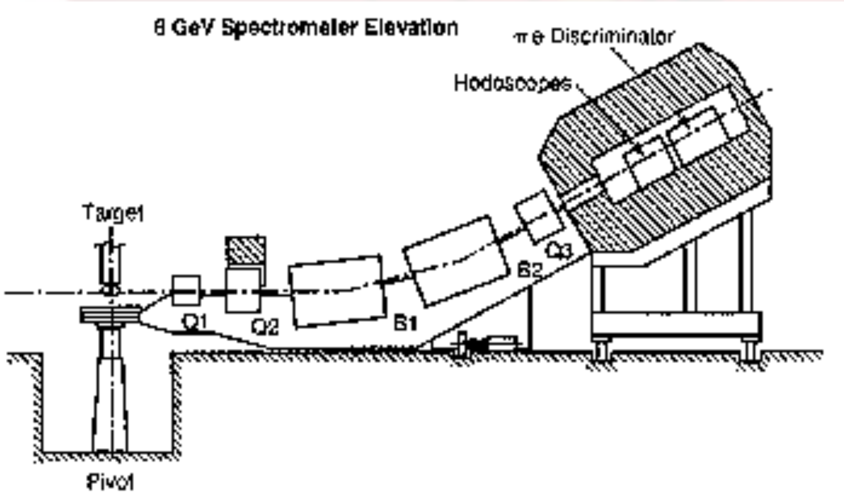
- Gell-Man Quark Model
  - Additional degree of freedom needed to satisfy the Fermi-Dirac statistics
  - Concept of color, Nc=3
- No free constituents found in experiments, Quark confinement
- Dynamics not yet understood then

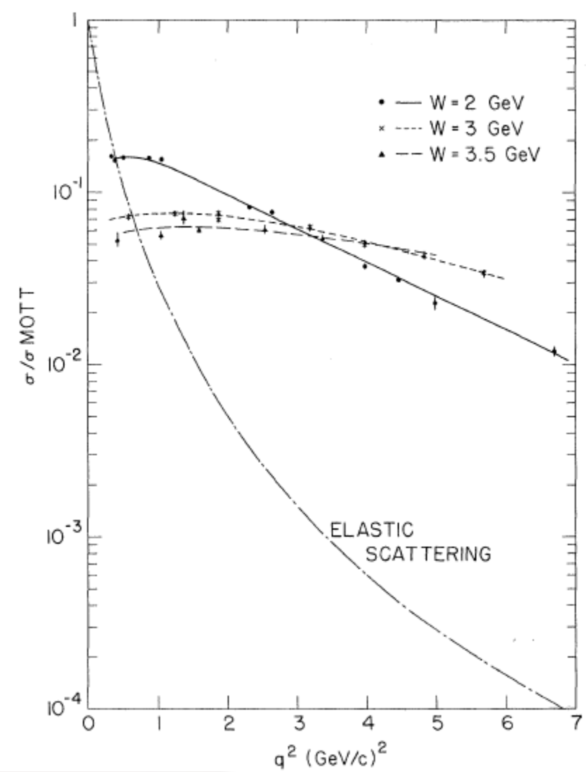
Deep Inelastic Scattering

Discovery of Quarks


Friedman Kendall Taylor





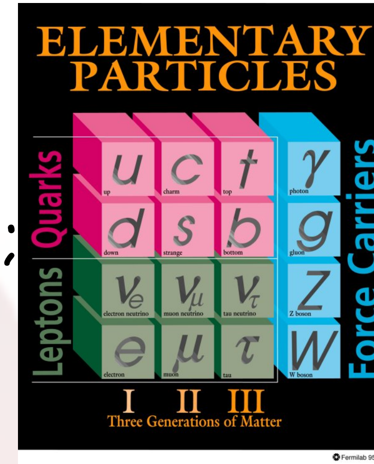


Bjorken Scaling: $Q^2 \rightarrow$ Infinity
Feynman Parton Model:
Point-like structure in Nucleon

Understanding the scaling

- Weak interactions at high momentum transfer
  - Rutherford formula rules
- Strong interaction at long distance
  - Form factors behavior
  - No free constituent found in experiment
- Strong interaction dynamics is different from previous theory

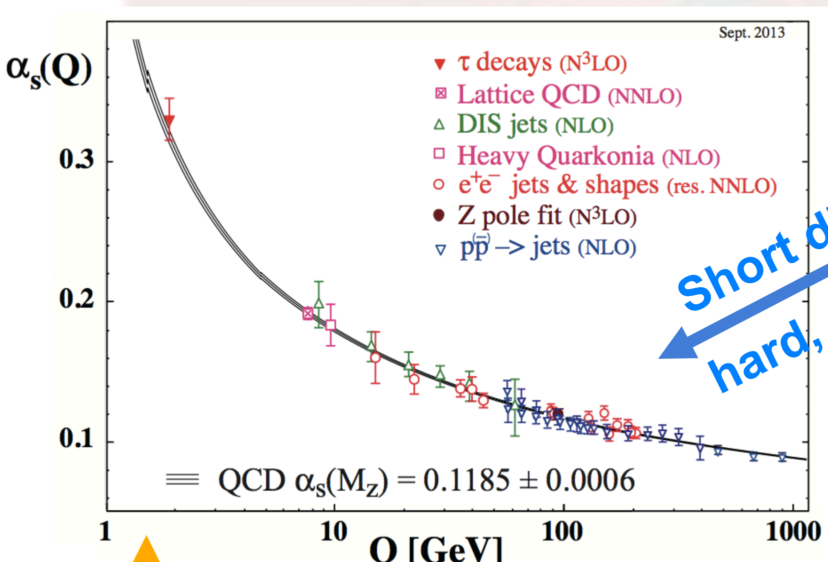
QCD and Strong-Interactions



- QCD: Non-Abelian gauge theory
  - Building blocks: quarks (spin½, m_q, 3 colors; gluons: spin 1, massless, 3^2-1 colors)

$$L = \bar{\psi}(i\gamma\cdot\partial - m_q)\psi - \frac{1}{4}F^{\mu\nu a}F_{\mu\nu a} - g_s\bar{\psi}\gamma\cdot A\psi$$

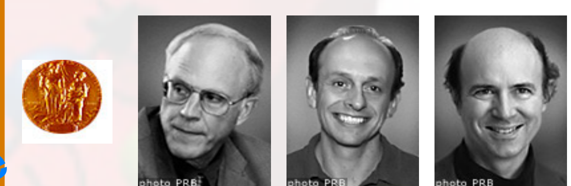
- Asymptotic freedom and confinement



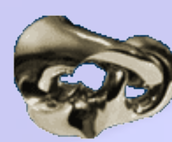
~1/Length

Nonperturbative scale Λ_{QCD}~1GeV

Short distance hard, perturbative



Long distance:? Soft, non-perturbative



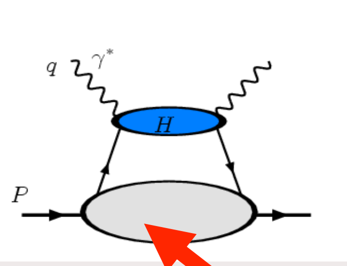
Clay Mathematics Institute Millennium Prize Problem

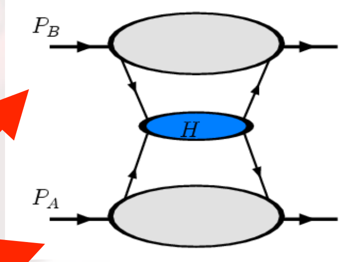
Quantum Chromodynamics

- There is no doubt that QCD is the right theory for hadron physics
- However, many fundamental questions…
- How does the **nucleon mass**?
- Why quarks and gluons are **confined** inside the nucleon?
- How do the fundamental **nuclear forces** arise from QCD?
- We don't have a **comprehensive picture** of the nucleon structure as we don't have an approximate QCD nucleon wave function
- …

Feynman's parton language and QCD Factorization

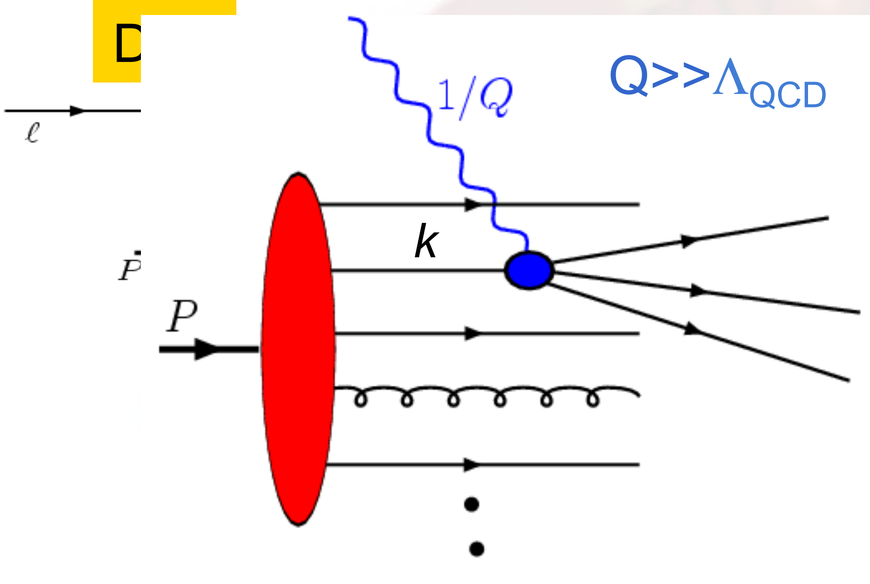
- If a hadron is involved in high-energy scattering, the physics simplifies in the infinite momentum frame (Feynman's Parton Picture)
- The scattering can be decomposed into a convolution of parton scattering and parton density (distribution), or wave function or correlations
  - QCD Factorization!

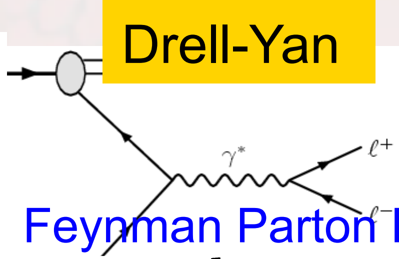




$$\sim \int \text{Parton Distributions} \otimes \text{Hard Partonic Cross Section}$$

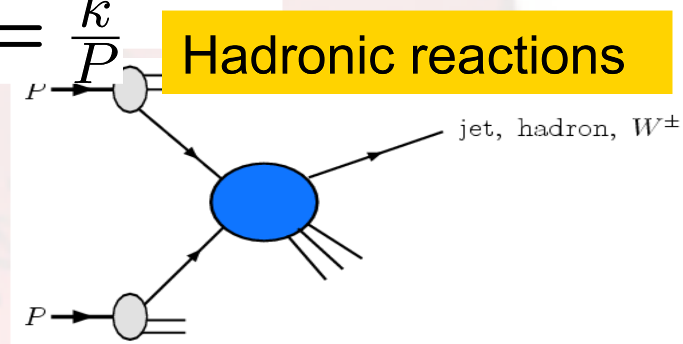
High energy scattering as a probe to the nucleon structure





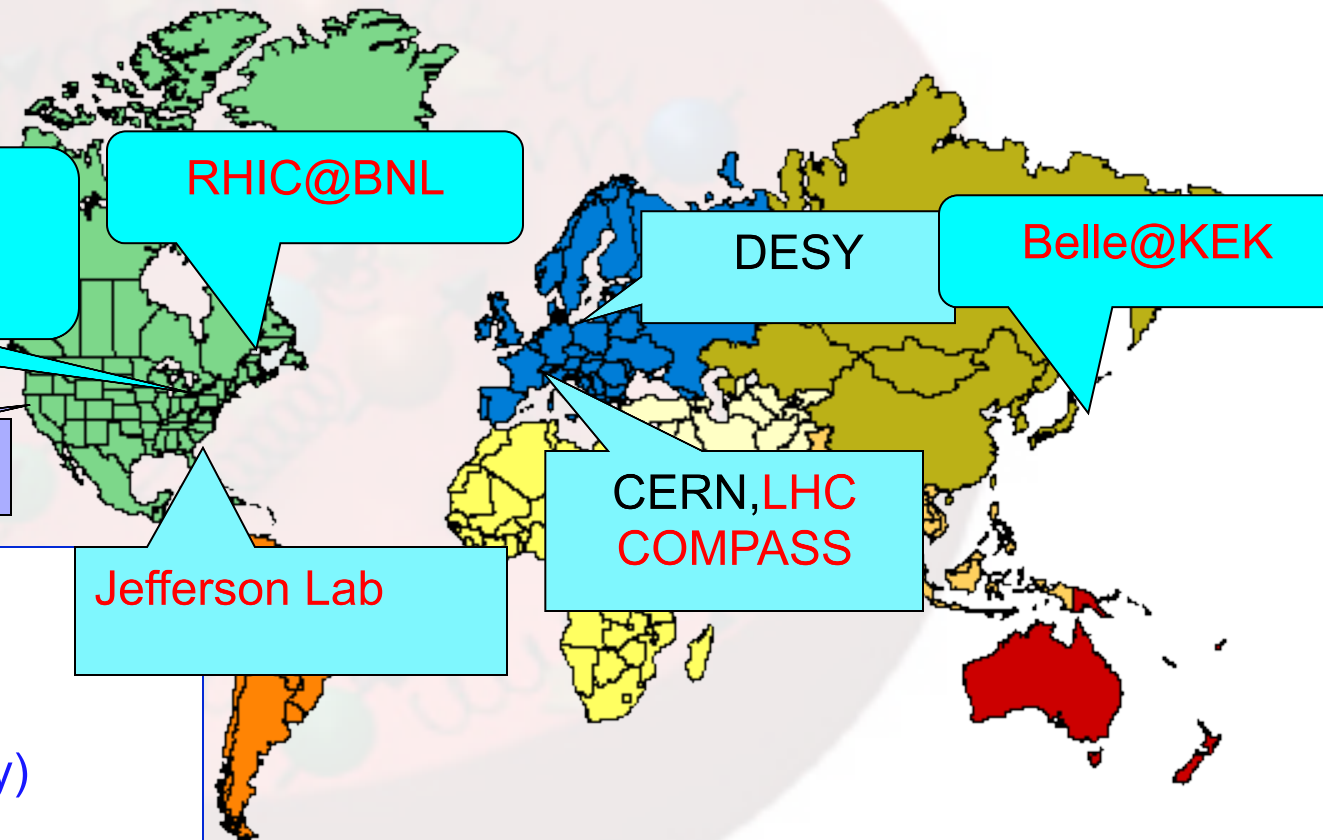
Feynman Parton Momentum fraction

$$x = \frac{k}{P}$$



- Many processes: Deep Inelastic Scattering, Deeply-virtual compton scattering, Drell-Yan lepton pair production, pp→jet+X
  - Momentum distribution: Parton Distribution
  - Spin density: polarized parton distribution
  - Wave function in infinite momentum frame: Generalized Parton Distributions

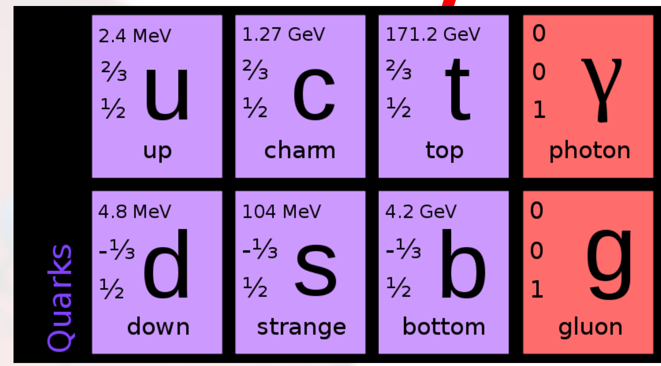
Exploring the partonic structure of nucleon worldwide



- Future facilities
  - Jlab@12GeV
  - JPARC (Japan)
  - GSI-FAIR (Germany)
  - **EIC, LHeC, …**

QCD: Nonabelian Gauge Theory

$$L = \bar{\psi}(i\gamma\cdot\partial - m_q)\psi - \frac{1}{4}F^{\mu\nu a}F_{\mu\nu a} - g_s\bar{\psi}\gamma\cdot A\psi$$



- Basic elements:
  - quark field, $\psi_{\rho a f}$, flavor, spin, color
  - gluon field, A_μ^α, color, spin
- Gauge transformation

$$\psi_{(0)\,\rho a f}(x) \mapsto \left[e^{-ig_0\omega_\alpha(x)t^\alpha}\right]_{ab}\psi_{(0)\,\rho b f}(x),$$

$$A^\alpha_{(0)\,\mu}(x)t^\alpha \mapsto \frac{-i}{g_0}e^{-ig_0\omega_\alpha(x)t^\alpha}D_\mu e^{ig_0\omega_\alpha(x)t^\alpha}$$

Non-abelian matrix

- Fundamental representation: the Gell-Man (3x3) matrix, T^a, (t^a), a=1,…,8
- Structure constant: $[t_\alpha, t_\beta] = i f_{\alpha\beta\gamma} t_\gamma$
  - $f_{\alpha\beta\gamma}$ are totally anti-symmetric
- Combination:

$$\mathrm{Tr}(t_\alpha t_\beta) = T_F \delta_{\alpha\beta},$$
$$t_\alpha t_\alpha = C_F I,$$
$$f_{\alpha\gamma\delta} f_{\beta\gamma\delta} = C_A \delta_{\alpha\beta},$$

Symbol	SU(n)	SU(3)
T_F	$\frac{1}{2}$	$\frac{1}{2}$
C_F	$\frac{n^2-1}{2n}$	$\frac{4}{3}$
C_A	n	3

Quantization

- Introduce the gauge fixing terms

$$\mathcal{L} = \bar{\psi}_0(i\not{D} - m_0)\psi_0 - \frac{1}{4}(G^{\alpha}_{(0)\,\mu\nu})^2 - \frac{1}{2\xi_0}(\partial \cdot A^{\alpha}_{(0)})^2$$
$$+ \partial_\mu \bar{\eta}_{0\,\alpha} \partial^\mu \eta_{0\,\alpha} + g_0 \partial_\mu \bar{\eta}_{0\,\gamma} f_{\alpha\beta\gamma} A^{\beta}_{(0)\,\mu} \eta_{0\,\alpha},$$

  - With a particular gauge: decouple the gluon field with the ghost, Physical Gauge
  - In covariant gauge, we have to include the (fermion scalar) ghost contribution
  - With this, we are ready to derive the Feynman rules

Feynman rules

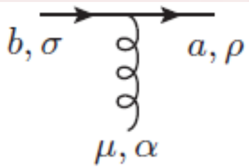
- Propagators

Gluon		$\left[-g_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2+i0}\right]\frac{i}{p^2+i0}$
Quark		$\frac{i(\not{p}+m_f)_{\rho\sigma}}{p^2-m_f^2+i0}$
Ghost		$\frac{i}{p^2+i0}$

Feynman rules

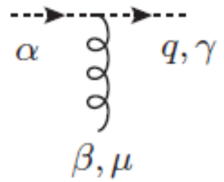
- Interaction vertices

Quark-gluon



$$-ig\mu^{\epsilon}(t^{\alpha})_{ab}\gamma^{\mu}_{\rho\sigma}$$

Ghost-gluon

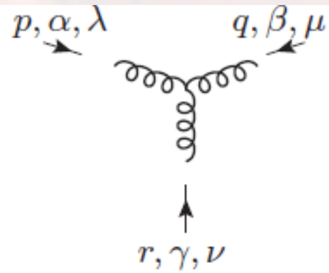


$$-g\mu^{\epsilon} f_{\alpha\beta\gamma} q^{\mu}$$

Feynman rules

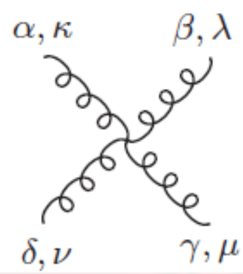
- Non-abelian part

Three-gluon

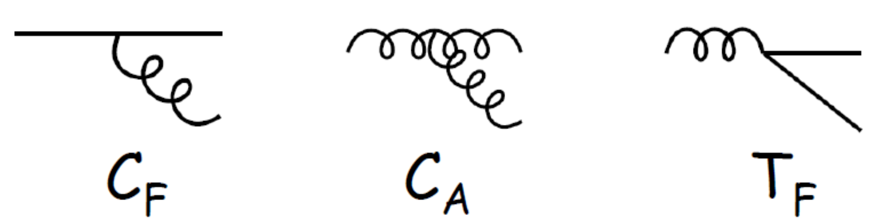


$$-g\mu^{\epsilon} f_{\alpha\beta\gamma}\left[(p-q)^{\nu} g^{\lambda\mu} + (q-r)^{\lambda} g^{\mu\nu} + (r-p)^{\mu} g^{\nu\lambda}\right]$$

Four-gluon

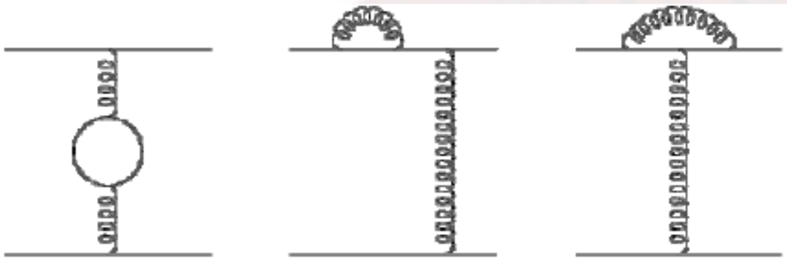


$$\begin{aligned}
&-ig^2\mu^{2\epsilon} f_{\epsilon\alpha\beta} f_{\epsilon\gamma\delta}(g^{\kappa\mu}g^{\lambda\nu} - g^{\kappa\nu}g^{\lambda\mu})\\
&-ig^2\mu^{2\epsilon} f_{\epsilon\alpha\gamma} f_{\epsilon\beta\delta}(g^{\kappa\lambda}g^{\mu\nu} - g^{\kappa\nu}g^{\lambda\mu})\\
&-ig^2\mu^{2\epsilon} f_{\epsilon\alpha\delta} f_{\epsilon\beta\gamma}(g^{\kappa\lambda}g^{\mu\nu} - g^{\kappa\mu}g^{\lambda\nu})
\end{aligned}$$



Screen and antiscreen

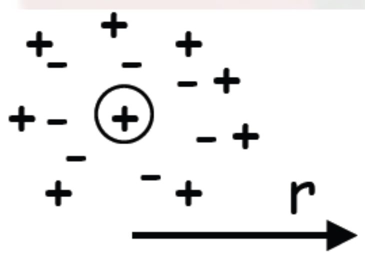
(coupling depends on distance)



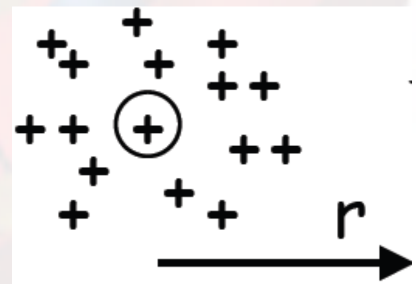
"screening" of the charge

"anti-screening"

QED-like



Non-abelian



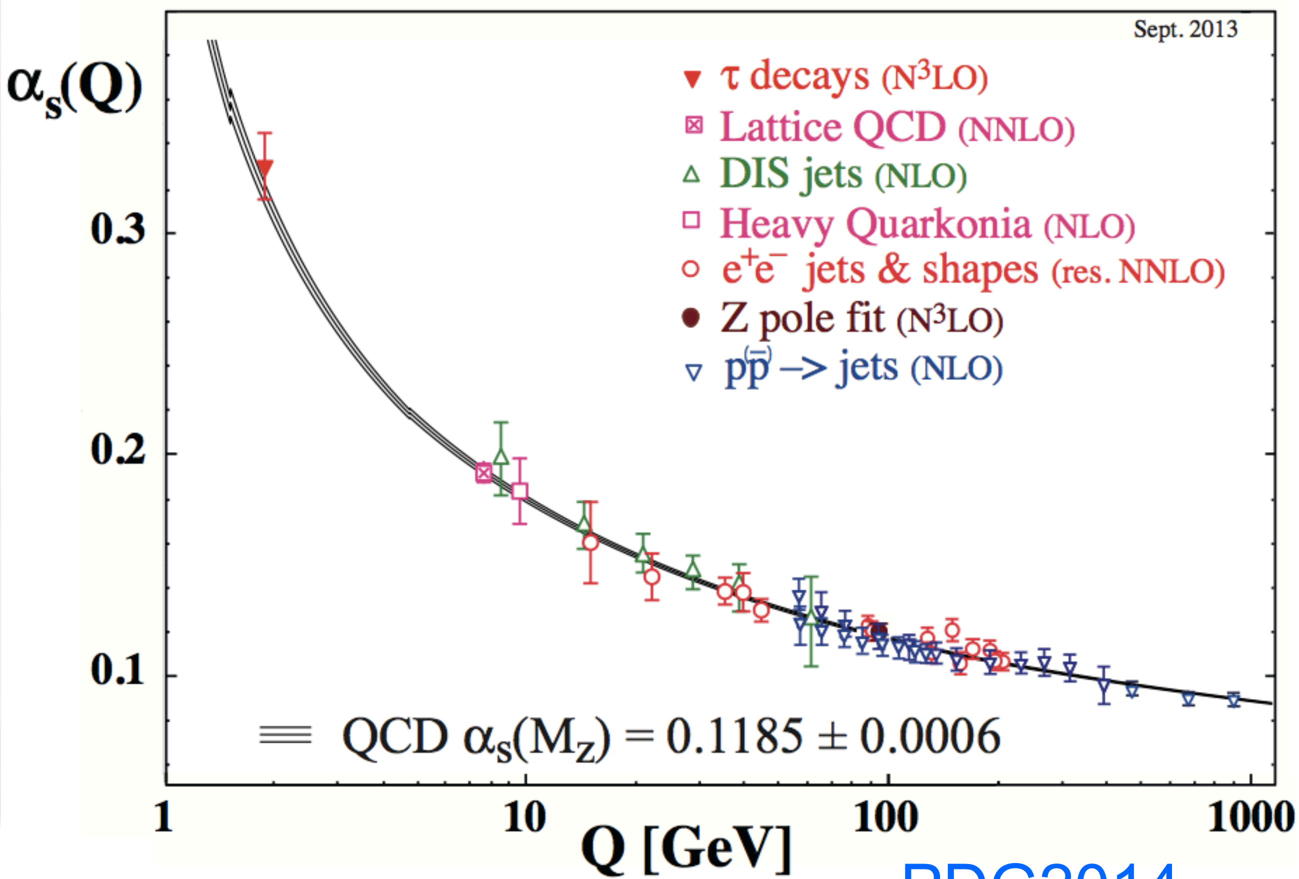
$$\frac{d\alpha_s/4\pi}{d\ln\mu} = 2\beta(\alpha_s/4\pi)$$

$$\beta(\alpha_s/4\pi) = -\left(\frac{11}{3}C_A - \frac{4}{3}T_F n_f\right)\frac{\alpha_s^2}{16\pi^2}$$

$$= -\left(11 - \frac{2}{3}n_f\right)\frac{\alpha_s^2}{16\pi^2}$$

Asymptotic freedom

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - \frac{2}{3}N_f)\ln(Q^2/\Lambda^2)}$$



PDG2014

Running of quark masses

- Same quark mass depends on the scale

$$m(\mu^2) \;=\; m({\mu_0}^2)\exp\left\{-\int_{\mu_0}^{\mu}\frac{d\lambda}{\lambda}\left[1+\gamma_m(g(\lambda))\right]\right\}$$

  - Vanishes as $\mu \to \infty$
- In practice, up, down, strange quarks can be treated as massless
  - u,d quarks: few MeV<<Λ; $m_s(\mu) \ll \mu$
- Charm, bottom, top quarks are heavy quarks
  - c: 1.5GeV, b: 5GeV, t: 170GeV

Perturbative corrections

- Singularities in higher order calculations
- Dimension regularization
  - n<4 for UV divergence
  - n>4 for IR divergence
  - MS ($\overline{\text{MS}}$) scheme for UV divergence

$$\int \frac{d^n k}{k^4} \to \int \frac{dk}{k} k^{n-4}$$

- pQCD predictions rely on Infrared safety of the particular calculation

pQCD predictions

- Infrared safe observables
  - Total cross section in e+e-→hadrons
  - EW decays, tau, Z, …
- Factorizable hard processes: parton distributions/fragmentation functions
  - Deep Inelastic Scattering
  - Drell-Yan Lepton pair production
  - Inclusive process in ep, ee, pp scattering, W, Higgs, jets, hadrons, …

- Light-cone wave functions, factorization for the hard exclusive processes
  - Generalized Parton Distributions and form factors
- Effective theory
  - Heavy quark effective theory, heavy meson decays
  - Non-relativistic QCD, heavy quarkonium decay and production
- Soft-collinear effective theory

Infrared safety (in general)

- Physics are not sensitive to the quark mass, not suffer from infrared divergence

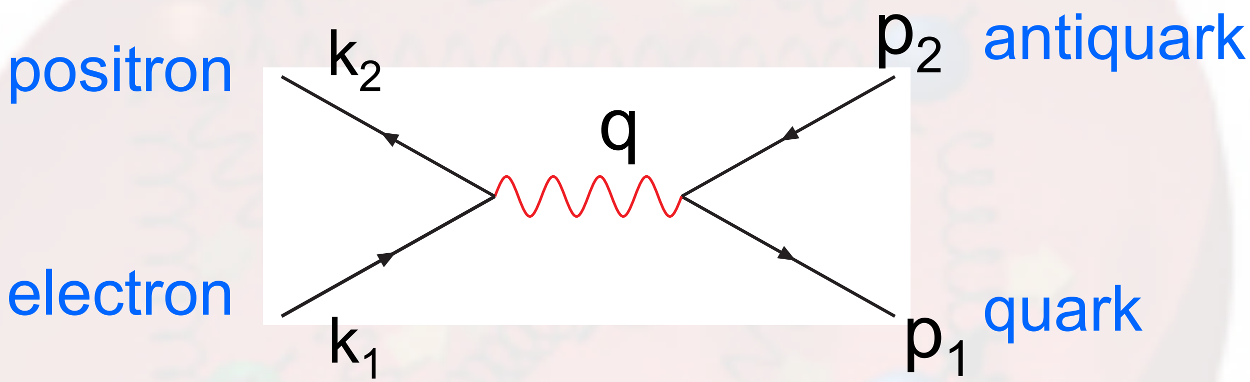
$$\tau\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\frac{m^2(\mu^2)}{\mu^2}\right)=\tau\left(1,\alpha_s(Q^2),\frac{m^2(Q^2)}{Q^2}\right)$$

- pQCD predictions rely on if we can obtain

$$\tau\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\frac{m^2(\mu^2)}{\mu^2}\right)\ \mu\overrightarrow{\rightarrow}\infty\quad \hat{\tau}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)+\mathcal{O}\left((\frac{m^2}{\mu^2})^a\right)$$

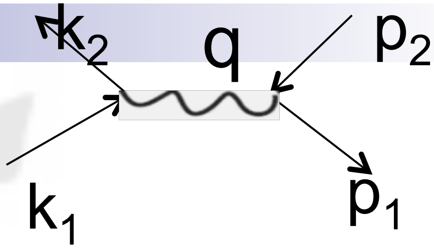
Simple example: e+e-→hadrons

- Leading order



  - Electron-positron annihilate into virtual photon, and decays into quark-antiquark pair, or muon pair
  - Quark-antiquark pair hadronize

Scattering amplitudes



$$B_\mu(p_1,p_2) = -iee_q\bar{u}_i(p_1)\gamma_\mu v_i(p_2), \quad A_\mu(k_1,k_2) = -ie\bar{v}(k_2)\gamma_\mu u(k_1)$$

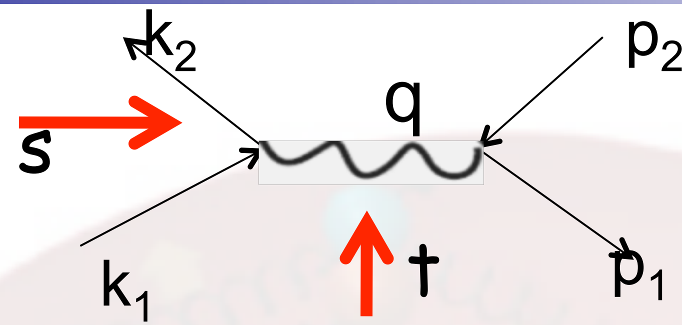
- i, color index for the quark, because photon doesnot carry color, the quark and antiquark have the same color
- e_q, the electric charge for the quark

$$L_{\mu\nu} = A_\mu A_\nu^* = 4e^2(k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu} - g_{\mu\nu}k_1 \cdot k_2)$$
$$H_{\mu\nu} = B_\mu B_\nu^* = 4e^2 e_q N_c(p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu} - g_{\mu\nu}p_1 \cdot p_2)$$

- Sum over the final state quark color → N_c
- Differential cross section $\frac{d\sigma}{d\Omega_{cm}} = \frac{1}{64\pi^2 E_{cm}^2}|\overline{\mathcal{M}}(e^+e^- \to q\bar{q})|^2$

$$|\overline{\mathcal{M}}|^2 = L_{\mu\nu}\frac{1}{q^4}H_{\mu\nu}^\dagger = N_c e_q^2 \frac{1}{4}\frac{e^4}{q^4} 32\left[k_1 \cdot p_1 k_2 \cdot p_2 + k_1 \cdot p_2 k_2 \cdot p1\right]$$

- ¼ comes from the average of the spin for initial leptons



- Some kinematics

$$s = (k_1 + k_2)^2 = (p_1 + p_2)^2 = E_{cm}^2 = q^2 = Q^2$$

$$t = (k_1 - p_1)^2 = (k_2 - p_2)^2 = -2k_1 \cdot p_1 = -\frac{s}{2}(1 - \cos\theta_{cm})$$

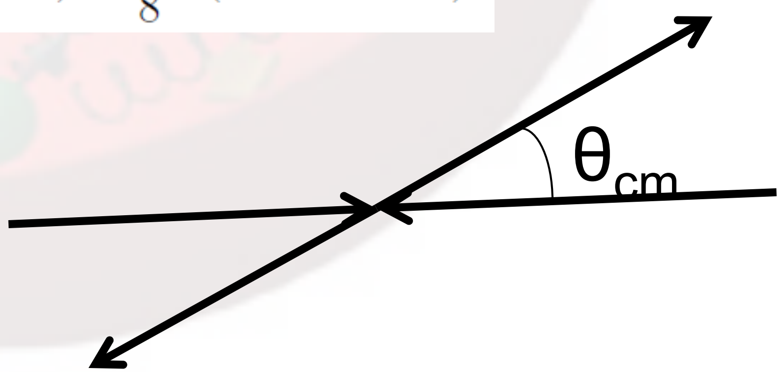
$$u = (k_1 - p_2)^2 = (k_2 - p_1)^2 = -2k_1 \cdot p_2 = -\frac{s}{2}(1 + \cos\theta_{cm})$$

- In the center of mass frame

$$k_1 \cdot p_1 k_2 \cdot p_2 + k_1 \cdot p_2 k_2 \cdot p1 = \frac{1}{4}(t^2 + u^2) = \frac{1}{8}s^2(1 + \cos^2\theta_{cm})$$

$$\frac{d\sigma}{d\Omega_{cm}} = N_c e_q^2 \frac{1}{4}\frac{\alpha^2}{Q^2}\left(1 + \cos^2\theta_{cm}^2\right)$$

$$\alpha = e^2/4\pi$$



- Total cross section

$$\sigma(e^+e^- \to q\bar{q}) = N_c \frac{4\pi}{3}\frac{\alpha^2}{Q^2} e_q^2$$

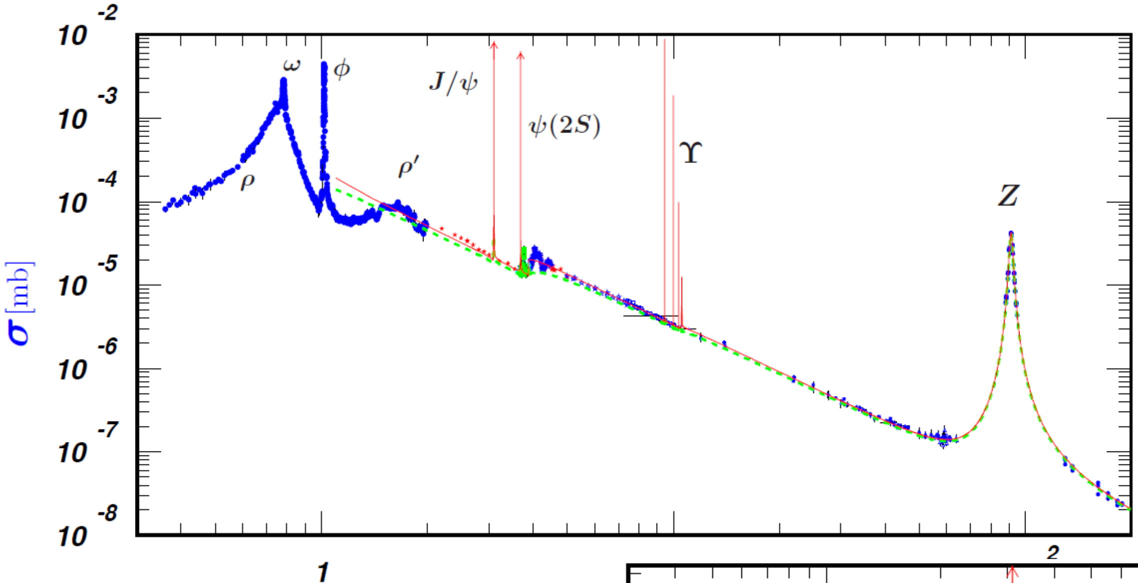
$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi}{3}\frac{\alpha^2}{Q^2}$$

- R ratio

$$R_{EW} = \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)} = N_c \sum e_q^2$$

  - Depends on the number of colors, electric charge of the quark

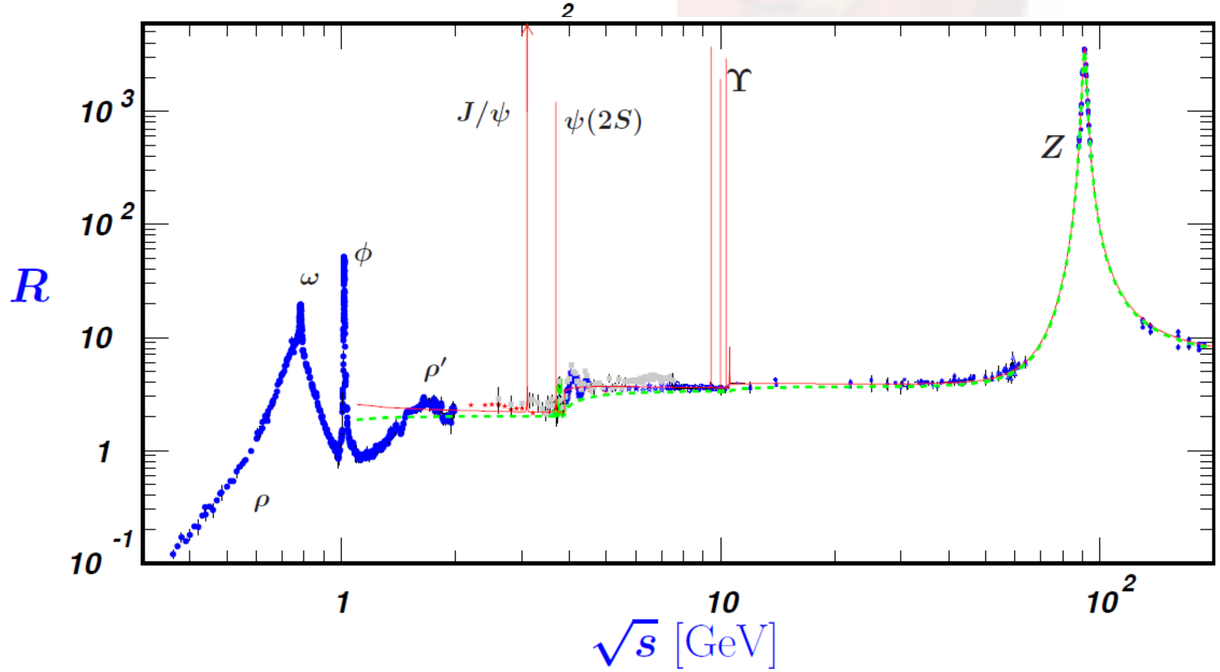
R ratio measurements

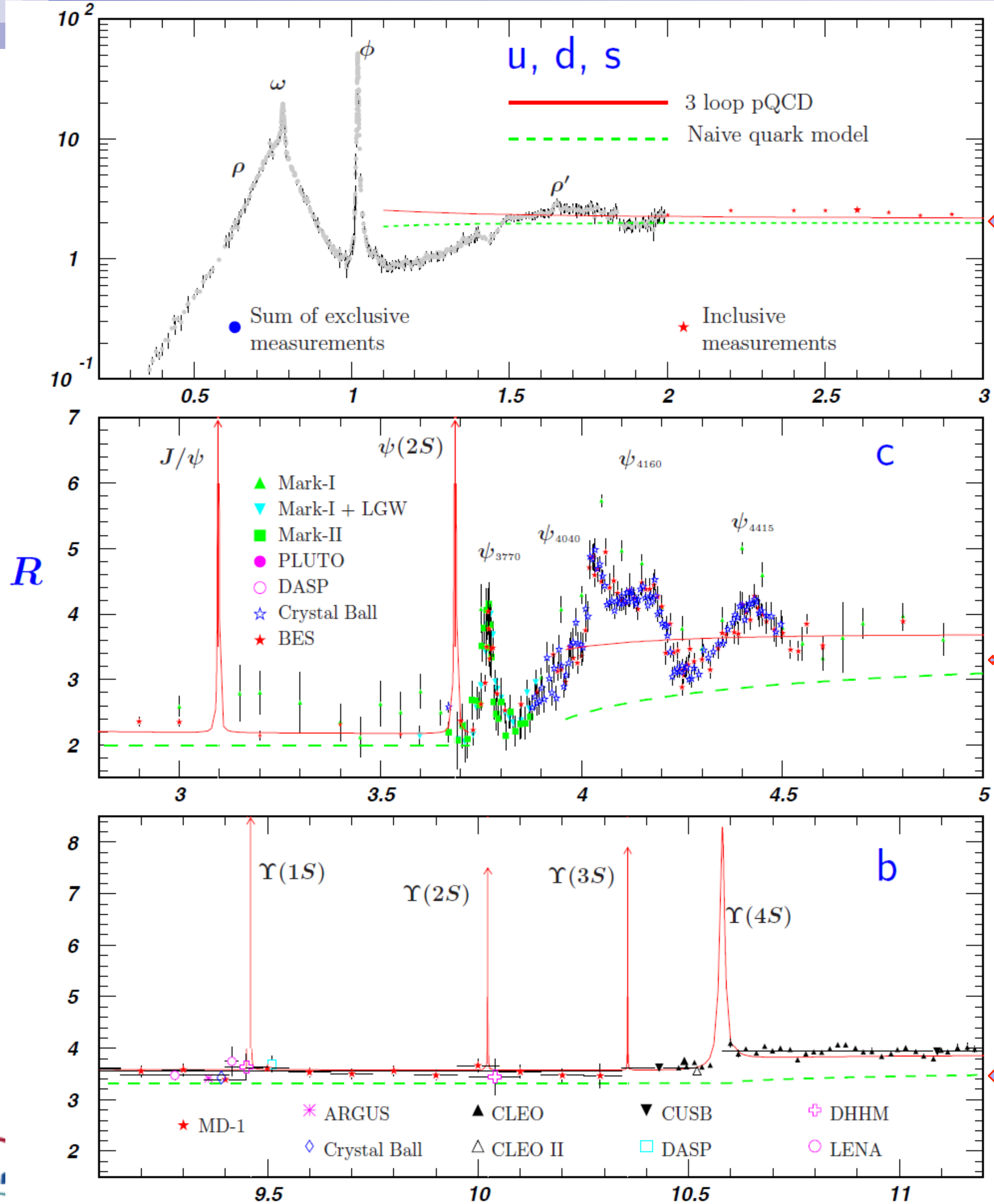


Hadronic cross sections

$e^+e^- \rightarrow$ hadrons

R-ratio





$N_c(e_u^2+e_d^2+e_s^2)=2$

$N_c(e_u^2+e_d^2+e_s^2+e_c^2)=2+4/3$

$N_c(e_u^2+e_d^2+e_s^2+e_c^2+e_b^2)$
$=2+5/3$

Perturbative corrections

- The total cross section is infrared safe

$$\frac{\sigma(e^+e^- \to \text{hadrons}, Q)}{\sigma(e^+e^- \to \mu^+\mu^-, Q)} \equiv R(Q) = R_{\text{EW}}(Q)(1 + \delta_{\text{QCD}}(Q))$$

$$\delta_{\text{QCD}}(Q) = \sum_{n=1}^{\infty} c_n \cdot \left(\frac{\alpha_s(Q^2)}{\pi}\right)^n + \mathcal{O}\left(\frac{\Lambda^4}{Q^4}\right)$$

$$c_1 = 1\,, \qquad c_2 = 1.9857 - 0.1152 n_f\,,$$

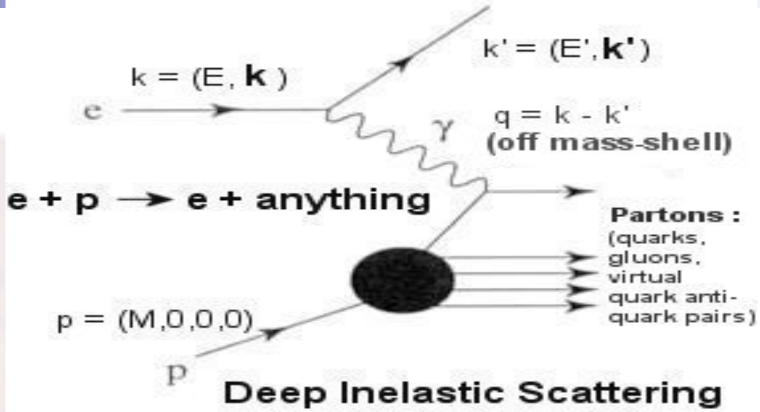
$$c_3 = -6.63694 - 1.20013 n_f - 0.00518 n_f^2 - 1.240\eta$$

$$c_4 = -156.61 + 18.77 n_f - 0.7974 n_f^2 + 0.0215 n_f^3 + C\eta\,, \quad \eta = (\sum e_q)^2/(3\sum e_q^2)$$

Long distance physics (factorization)

- Not every quantities calculated in perturbative QCD are infrared safe
  - Hadrons in the initial/final states, e.g.
- Factorization guarantee that we can safely separate the long distance physics from short one
- There are counter examples where the factorization does not work

Back to DIS



- Kinematics

$\nu = \frac{q \cdot P}{M} = E - E'$ is the lepton's energy loss in the nucleon rest frame (in earlier literature sometimes $\nu = q \cdot P$). Here, E and E' are the initial and final lepton energies in the nucleon rest frame.

$Q^2 = -q^2 = 2(EE' - \vec{k} \cdot \vec{k}') - m_\ell^2 - m_{\ell'}^2$ where $m_\ell(m_{\ell'})$ is the initial (final) lepton mass. If $EE' \sin^2(\theta/2) \gg m_\ell^2, m_{\ell'}^2$, then

$\approx 4EE' \sin^2(\theta/2)$, where θ is the lepton's scattering angle with respect to the lepton beam direction.

$x = \frac{Q^2}{2M\nu}$ where, in the parton model, x is the fraction of the nucleon's momentum carried by the struck quark.

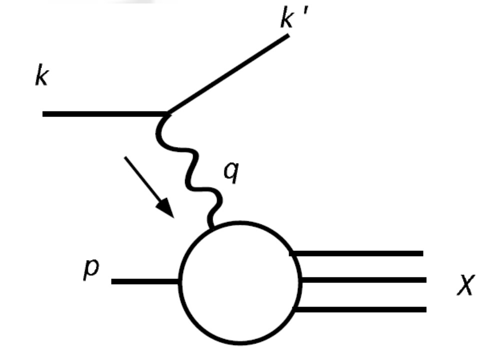
$y = \frac{q \cdot P}{k \cdot P} = \frac{\nu}{E}$ is the fraction of the lepton's energy lost in the nucleon rest frame.

$W^2 = (P + q)^2 = M^2 + 2M\nu - Q^2$ is the mass squared of the system X recoiling against the scattered lepton.

$s = (k + P)^2 = \frac{Q^2}{xy} + M^2 + m_\ell^2$ is the center-of-mass energy squared of the lepton-nucleon system.

BERKELEY

Structure functions (cross section)



- EM factorization (photon exchange)

$$d\sigma = \frac{d^3k'}{2s|\vec{k}'|}\frac{1}{(q^2)^2}L^{\mu\nu}(k,q)W_{\mu\nu}(p,q) \qquad L^{\mu\nu} \equiv \frac{e^2}{8\pi^2}tr\left[\not{k}\gamma^\mu \not{k}'\gamma^\nu\right]$$

- Hadronic tensor

$$W_{\mu\nu} \equiv \frac{1}{8\pi}\sum_{\text{spins }\sigma}\sum_X < N(p,\sigma) \mid J_\mu(0) \mid X > < X \mid J_\nu(0) \mid N(p,\sigma) > \times (2\pi)^4\delta^4(p_X - q - p)\,.$$

- Symmetry property for hadronic tensor
  - Spin average $W_{\mu\nu}^{(\mathrm{em})} = W_{\nu\mu}^{(\mathrm{em})}$
  - Time-reversal invariance $W_{\mu\nu} = W_{\mu\nu}^{*}$
  - Current conservation $q^{\mu} W_{\mu\nu} = 0$
  - Two independent structure functions

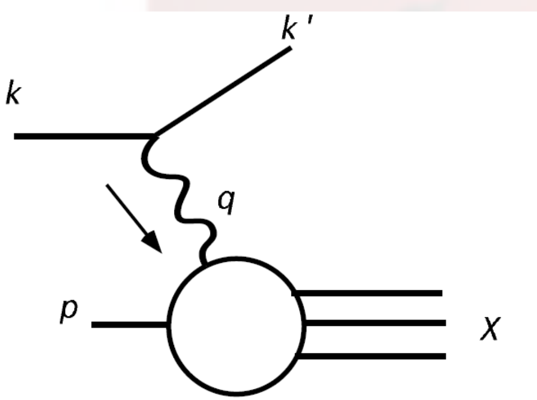
$$
\begin{aligned}
W_{\mu\nu}{}^{(em)} &= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1(x,q^2) \\
&\quad + \left(p_\mu + q_\mu \left(\frac{1}{2x}\right)\right)\left(p_\nu + q_\nu \left(\frac{1}{2x}\right)\right) W_2(x,q^2)
\end{aligned}
$$

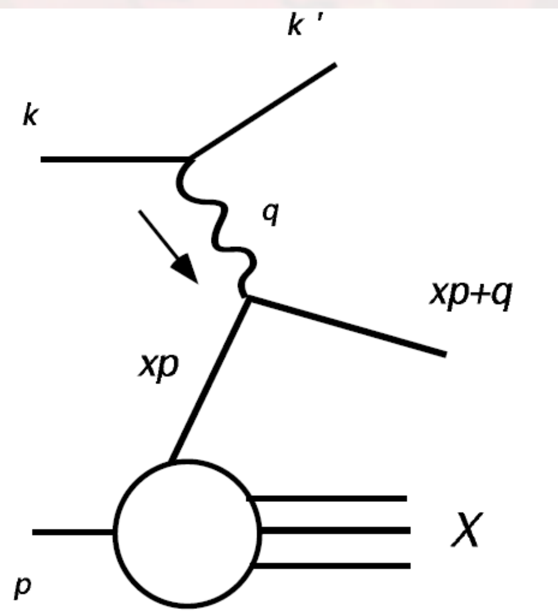
$$
= \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \frac{\hat{P}_\mu \hat{P}_\nu}{P\cdot q} F_2(x,Q^2) \qquad \hat{P}_\mu = P_\mu - \frac{P\cdot q}{q^2} q_\mu
$$

Naïve Parton Model

$$d\sigma^{(\ell N)}(p,q) = \sum_f \int_0^1 d\xi \; d\sigma_{\text{Born}}{}^{(\ell f)}(\xi p, q)\phi_{f/N}(\xi)$$

- $\phi_{f/N}(\xi)$ the parton distribution describes the probability that the quark carries nucleon momentum fraction





Partonic cross section eq→e'q'

- Cross symmetry with e+e-→qq

$$d\sigma = \frac{d^3k'}{2s|\vec{k'}|}\frac{1}{(q^2)^2}L^{\mu\nu}(k,q)W_{\mu\nu}(p,q) \qquad L^{\mu\nu} \equiv \frac{e^2}{8\pi^2}tr\left[\not{k}\gamma^\mu\not{k}'\gamma^\nu\right]$$

$$|\overline{\mathcal{M}}|^2 = \frac{1}{(q^2)^2}L_{\mu\nu}W_{\mu\nu} = e_q^2\frac{e^4}{(q^2)^2}2\left[s^2+u^2\right]$$

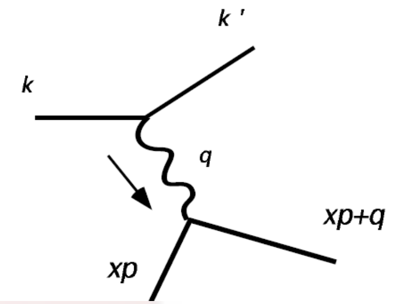
$$u = (k'-p)^2 = -2k'\cdot p = -s(1-y), \quad y = \frac{q\cdot p}{k\cdot p}$$

$$(s^2+u^2) = s^2(1+(1-y)^2)$$

$$d\sigma(ep\to e'+X) = \int dxdy\frac{2\pi\alpha^2}{Q^2}\left[1+(1-y)^2\right]\sum_q e_q^2\phi_{q/P}(x)$$

Naïve Parton Model

$$d\sigma^{(\ell N)}(p,q) = \sum_f \int_0^1 d\xi \; d\sigma_{\text{Born}}{}^{(\ell f)}(\xi p, q)\phi_{f/N}(\xi)$$



- Partonic tensor is calculated

$$W_{\mu\nu}{}^{(f)} = \frac{1}{8\pi}\int \frac{d^3p'}{(2\pi)^3 2\omega_{p'}} Q_f{}^2 tr[\gamma_\mu \not{p}' \gamma_\nu \not{p}]\,(2\pi)^4\delta^4(p' - \xi p - q)$$

- Structure functions

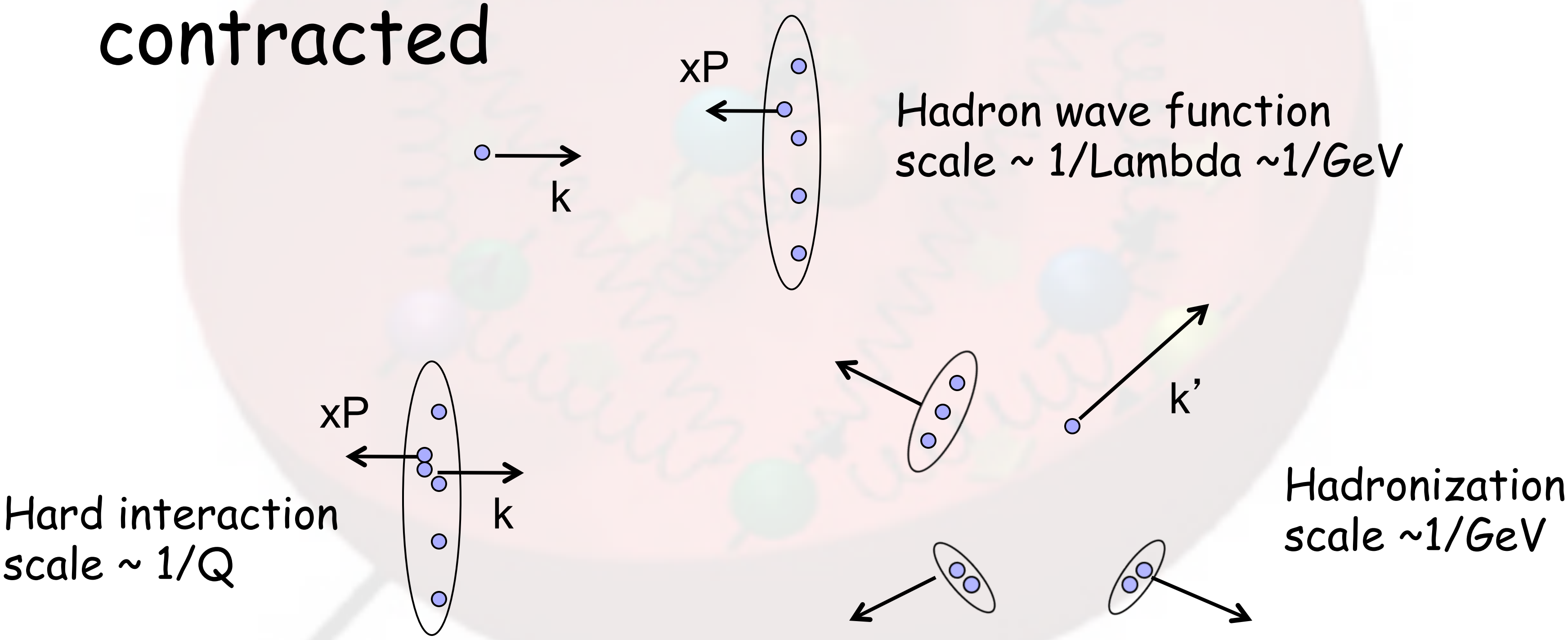
$$F_2{}^{(N)}(x) = \sum_f Q_f{}^2 x\phi_{f/N}(x) = 2xF_1{}^{(N)}(x)$$

  - Callan-Gross relation:
  - Quark spin is $\frac{1}{2}$

$$F_2 = 2xF_1$$

Intuitive argument for the factorization (DIS)

- In the Bjorken limit, nucleon is Lorentz contracted



Factorization formula

$$F_2^{(h)}(x,Q^2) = \sum_{i=f,\bar f,G} \int_x^1 d\xi\, C_2^{(i)}\left(\frac{x}{\xi},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)\phi_{i/h}(\xi,\mu^2)$$

$$F_1^{(h)}(x,Q^2) = \sum_{i=f,\bar f,G} \int_x^1 \frac{d\xi}{\xi}\, C_1^{(i)}\left(\frac{x}{\xi},\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)\phi_{i/h}(\xi,\mu^2)$$

- Factorization → scale dependence

$$\mu\frac{d^2}{d\mu^2}\phi_{i/h}(x,\mu^2) = \sum_{j=f,\bar f,G}\int_x^1 \frac{d\xi}{\xi} P_{ij}\left(\frac{x}{\xi},\alpha_s(\mu^2)\right)\phi_{j/h}(\xi,\mu^2)$$

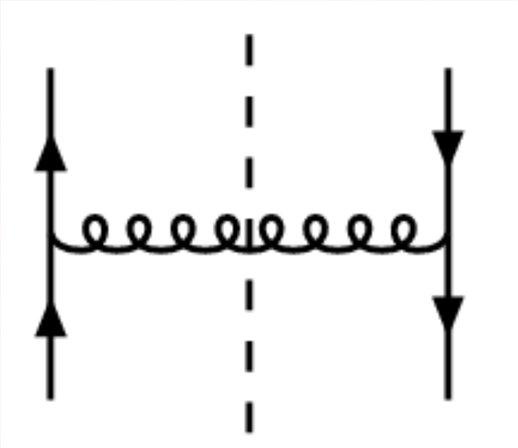
$$\mu\frac{d}{d\mu}\ln\bar\phi\left(n,\alpha_s(\mu^2)\right) = -\gamma_n\left(\alpha_s(\mu^2)\right) \qquad \bar f(n) \equiv \int_0^1 dx\, x^{n-1} f(x)$$

- Scale dependence → resummation

$$\bar\phi^{(\mathrm{val})}(n,\mu^2) = \bar\phi^{(\mathrm{val})}(n,\mu_0^2)\exp\left\{-\frac{1}{2}\int_0^{\ln\mu^2/\mu_0^2} dt\,\gamma_n\left(\alpha_s(\mu_0^2 e^t)\right)\right\}$$

anomalous dimension: $\int_0^1 d\xi\,\xi^{n-1}P_{ij}(\xi,\alpha_s) = -\gamma_{ij}(n)$

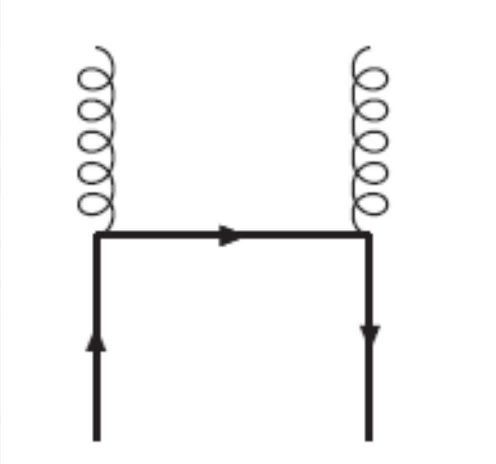
Quark-quark splitting



- Physical polarization for the radiation gluon
- Incoming quark on-shell, outgoing quark off-shell

$$\mathcal{P}_{qq} = C_F \left[\frac{1+x^2}{(1-x)_+} + \delta(1-x) \right]$$

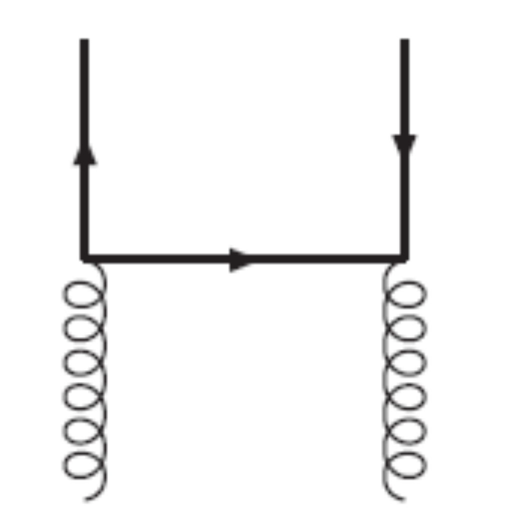
Quark-gluon splitting



- Incoming quark on-shell, gluon is off-shell

$$\mathcal{P}_{g/q} = C_F \left[\frac{1 + (1-x)^2}{x} \right]$$

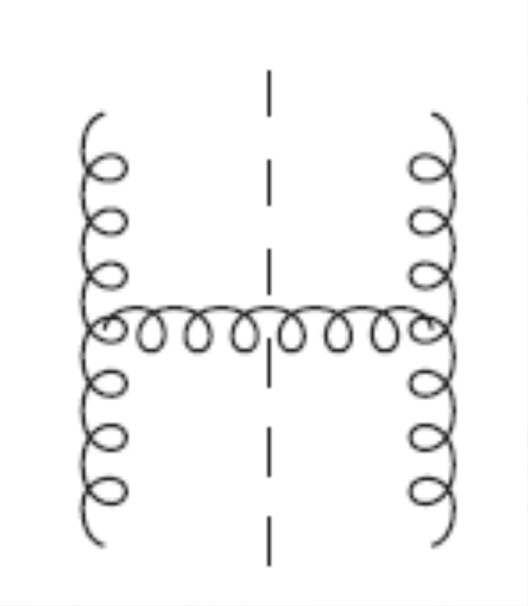
Gluon-quark splitting



- Incoming gluon is on-shell, physical polarization

$$\mathcal{P}_{q/g} = T_F \left[(1-x)^2 + x^2 \right]$$

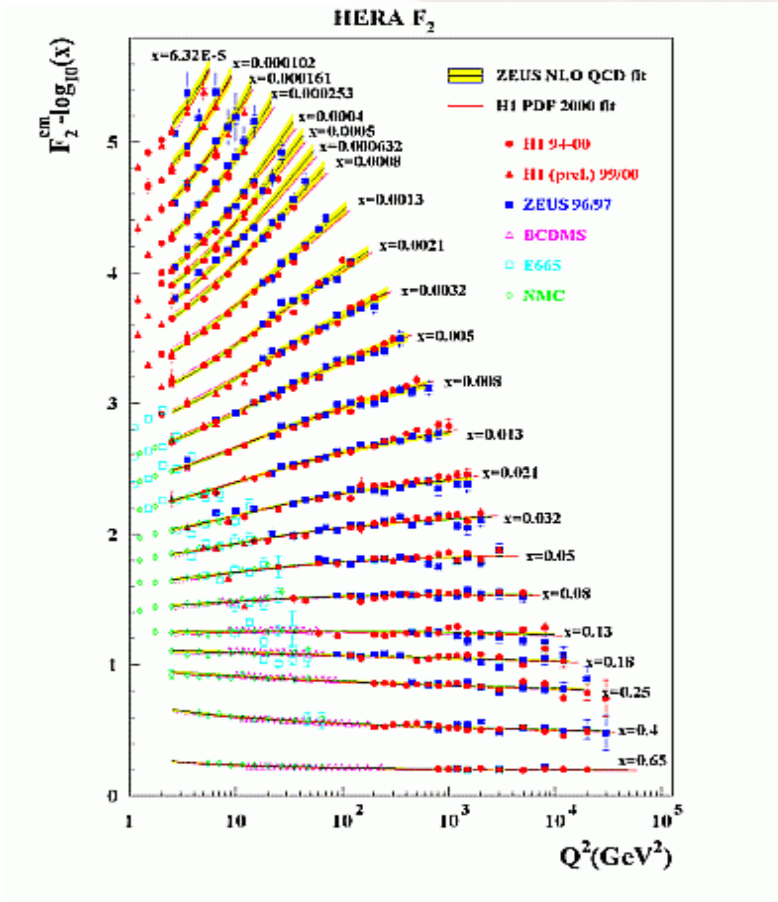
Gluon-gluon splitting

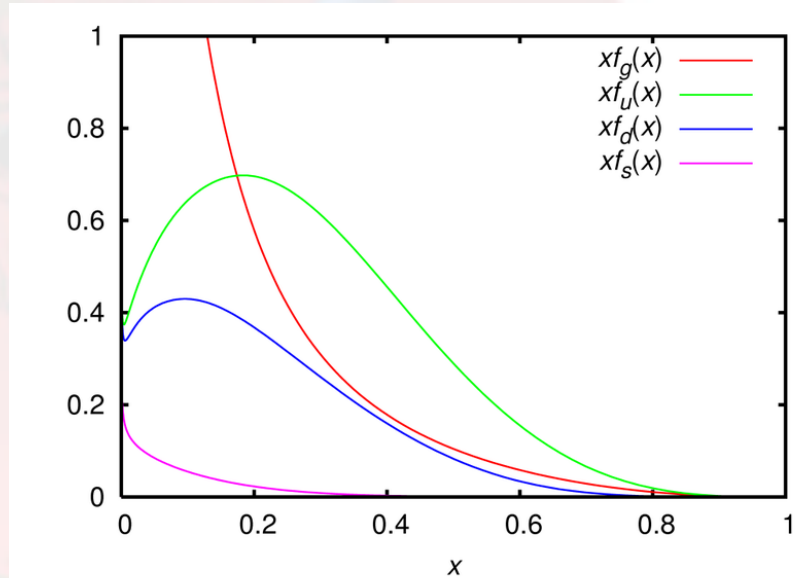


- Physical polarizations for the gluons

$$\mathcal{P}_{gg}(x) = \frac{x}{(1-x)_+} + \frac{1-x}{x} + x(1-x) + \delta(x-1)\beta_0$$

These evolutions describe the HERA data





CTEQ6

Reverse the DIS: Drell-Yan

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

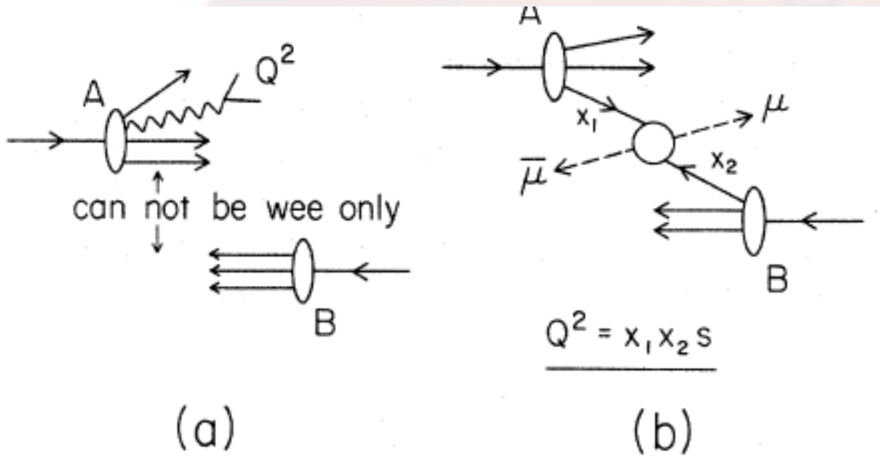
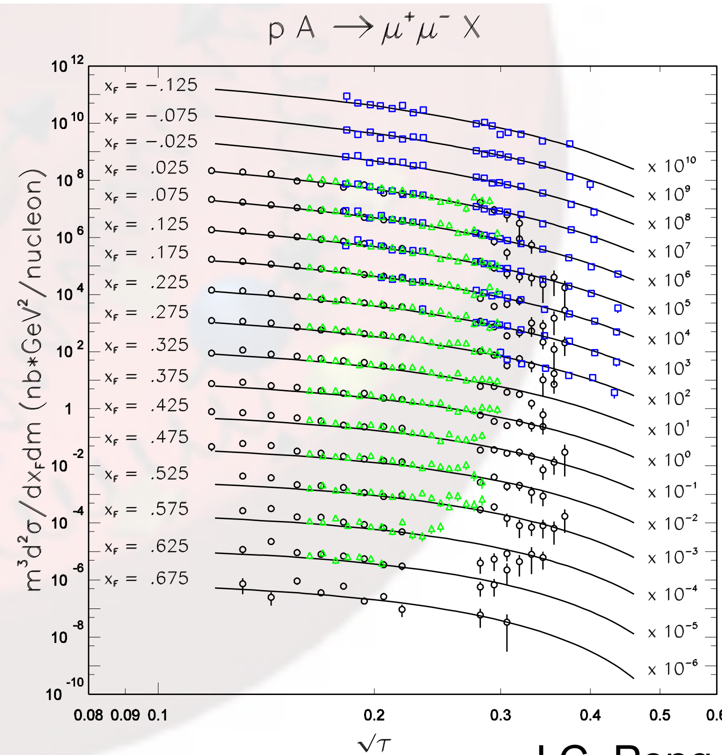
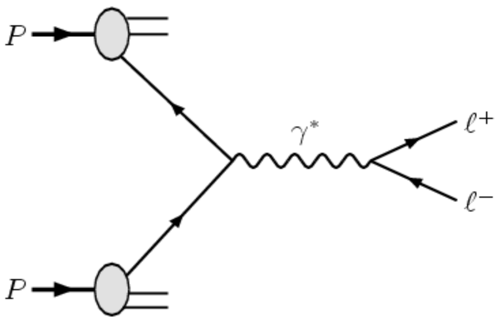


FIG. 1. (a) Production of a massive pair Q^2 from one of the hadrons in a high-energy collision. In this case it is kinematically impossible to exchange "wee" partons only. (b) Production of a massive pair by parton-antiparton annihilation.



J.C. Peng

Drell-Yan lepton pair production



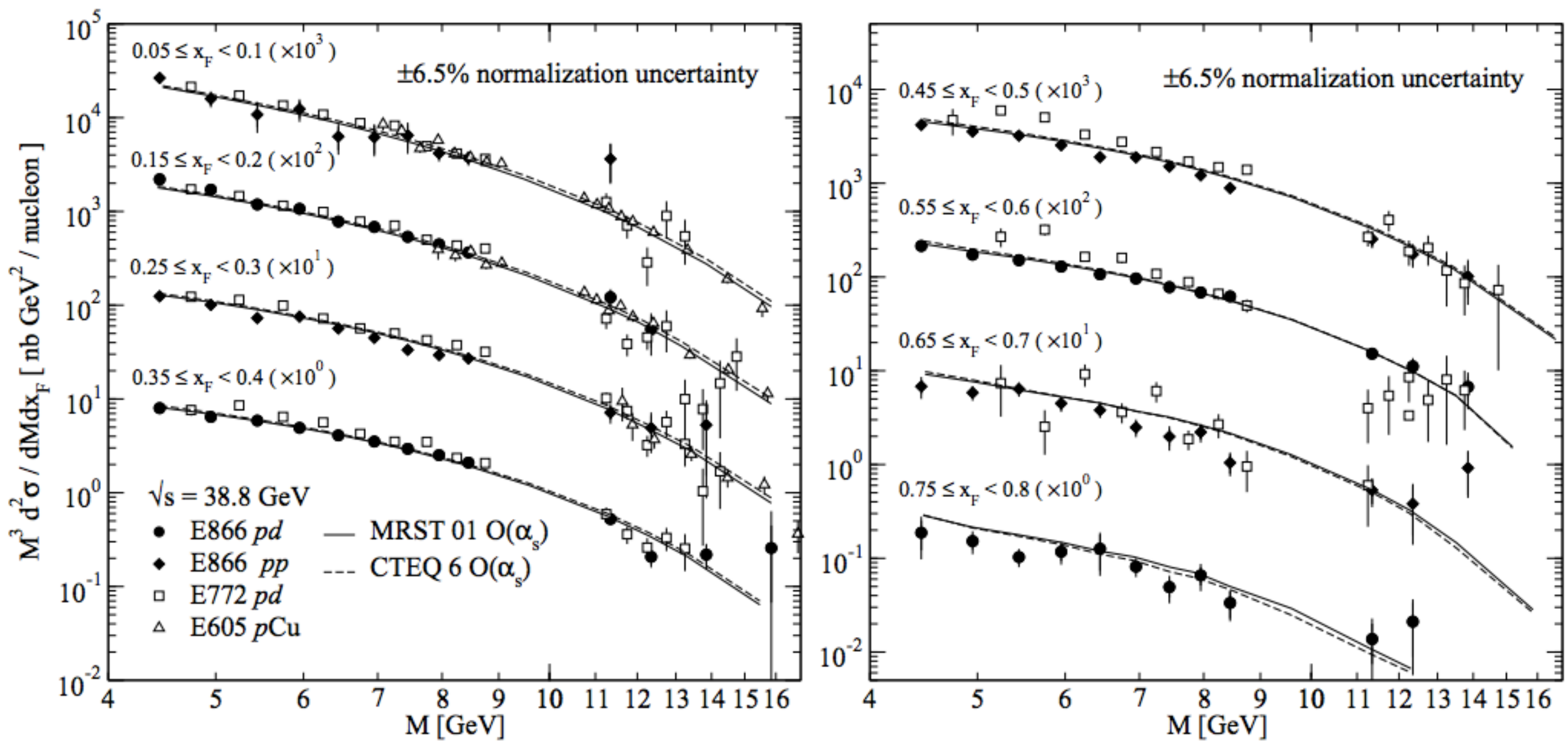
$$\sigma(pp \to \ell^+\ell^- + X) = \int dx_1 dx_2 \phi_{q/p}(x_1)\phi_{\bar{q}/p}(x_2)\hat{\sigma}(q\bar{q} \to \ell^+\ell^-)$$

- The same parton distributions as DIS
  - Universality
- Partonic cross section

$$\sigma(e^+e^- \to q\bar{q}) = N_c \frac{4\pi}{3}\frac{\alpha^2}{Q^2}e_q^2$$

$$\hat{\sigma}(q\bar{q} \to \ell^+\ell^-) = \frac{4\pi}{3}\frac{\alpha^2}{Q^2}e_q^2\frac{1}{N_c}$$

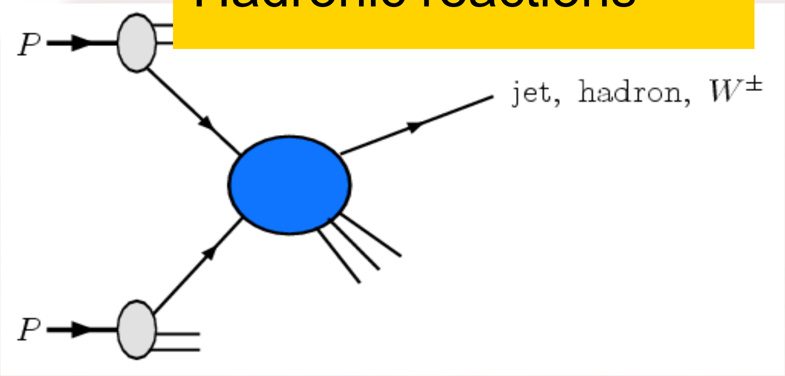
Profound results



- Universality
- Perturbative QCD at work

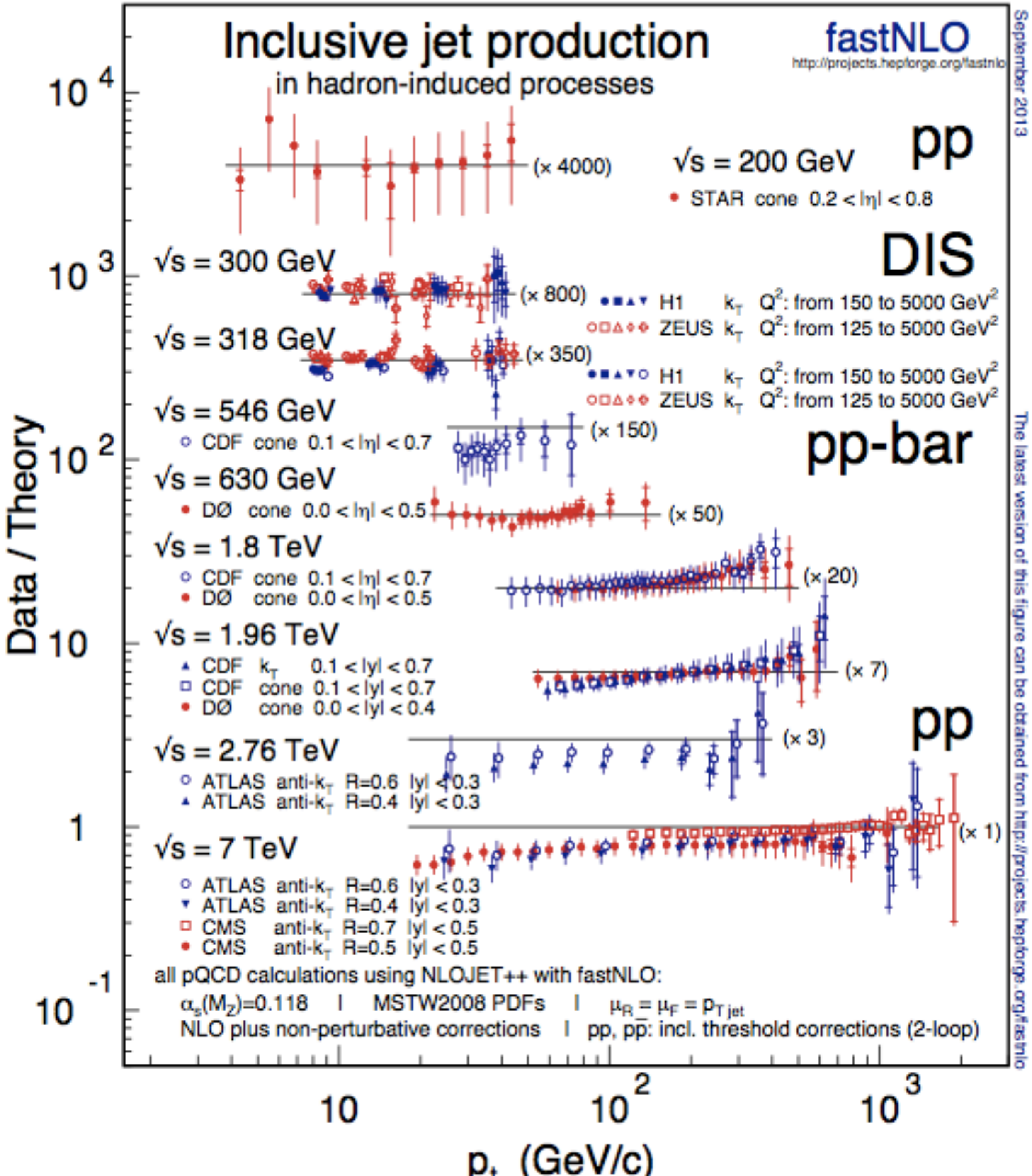
More general hadronic process

Hadronic reactions



$$\sigma(pp \to c + X) = \int dx_1 dx_2 \phi_{a/p}(x_1)\phi_{b/p}(x_2)\hat{\sigma}(ab \to c + X)$$

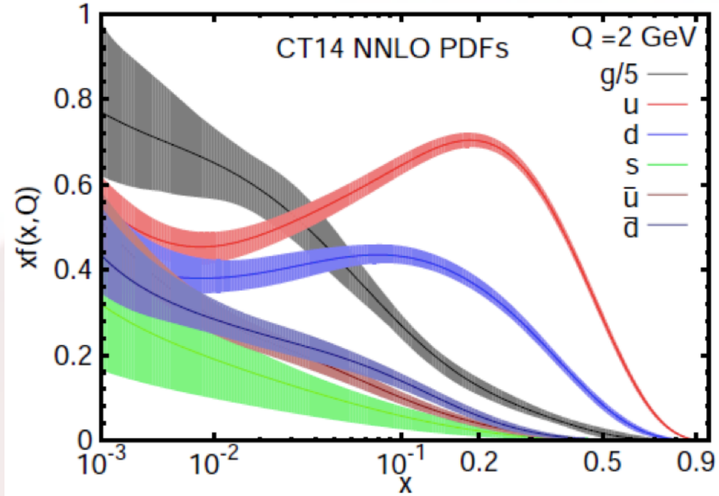
- All these processes have been computed up to next-to-leading order, some at NNLO, few at N^3LO



Inclusive jet production
in hadron-induced processes
fastNLO
http://projects.hepforge.org/fastnlo
September 2013
pp
√s = 200 GeV
STAR cone 0.2 < |η| < 0.8
(× 4000)
DIS
√s = 300 GeV
(× 800)
H1 k_T Q²: from 150 to 5000 GeV²
ZEUS k_T Q²: from 125 to 5000 GeV²
√s = 318 GeV
(× 350)
H1 k_T Q²: from 150 to 5000 GeV²
ZEUS k_T Q²: from 125 to 5000 GeV²
pp-bar
√s = 546 GeV
CDF cone 0.1 < |η| < 0.7
(× 150)
√s = 630 GeV
DØ cone 0.0 < |η| < 0.5
(× 50)
√s = 1.8 TeV
CDF cone 0.1 < |η| < 0.7
DØ cone 0.0 < |η| < 0.5
(× 20)
√s = 1.96 TeV
CDF k_T 0.1 < |y| < 0.7
CDF cone 0.1 < |y| < 0.7
DØ cone 0.0 < |y| < 0.4
(× 7)
pp
√s = 2.76 TeV
ATLAS anti-k_T R=0.6 |y| < 0.3
ATLAS anti-k_T R=0.4 |y| < 0.3
(× 3)
√s = 7 TeV
ATLAS anti-k_T R=0.6 |y| < 0.3
ATLAS anti-k_T R=0.4 |y| < 0.3
CMS anti-k_T R=0.7 |y| < 0.5
CMS anti-k_T R=0.5 |y| < 0.5
(× 1)
all pQCD calculations using NLOJET++ with fastNLO:
α_s(M_Z)=0.118 | MSTW2008 PDFs | μ_R = μ_F = p_T jet
NLO plus non-perturbative corrections | pp, pp̄: incl. threshold corrections (2-loop)
The latest version of this figure can be obtained from http://projects.hepforge.org/fastnlo
Data / Theory
p_t (GeV/c)

PDG2014

Parton picture of the nucleon

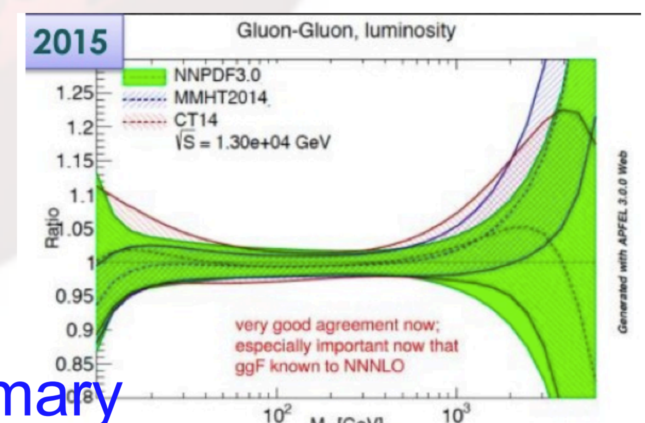


C.-P.Yuan@DIS15

- Beside valence quarks, there are sea and gluons
- Precisions on the PDFs are very much relevant for LHC physics: SM/New Physics

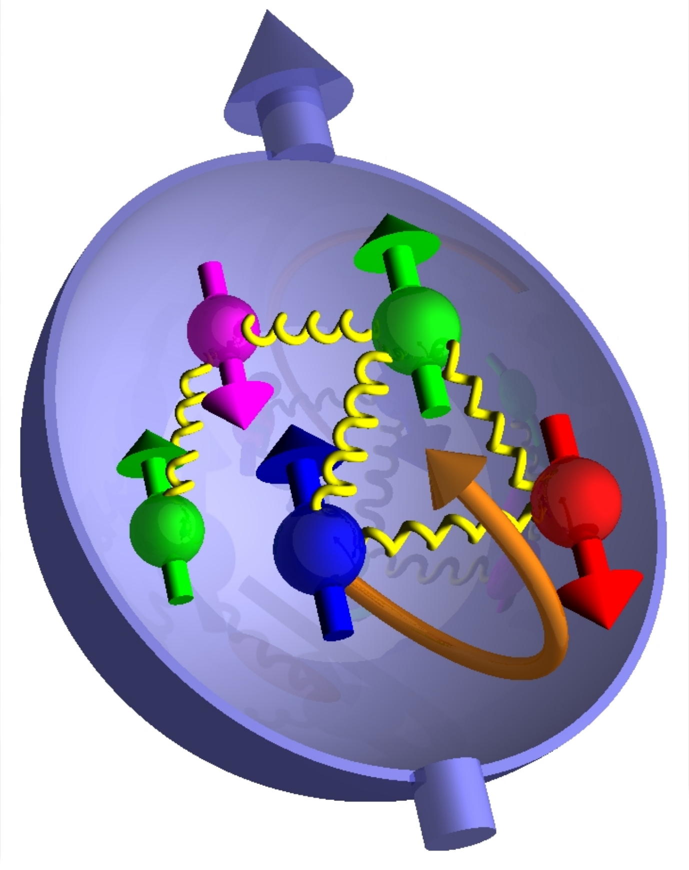
$$\sigma(gg \to H), \sqrt(s) = 13\text{TeV}$$

CT14	MMHT2014	NNPDF3.0
42.68 pb	42.70 pb	42.97 pb
+2.0% -2.4%	+1.3% -1.8%	+1.9% -1.9%



DIS summary

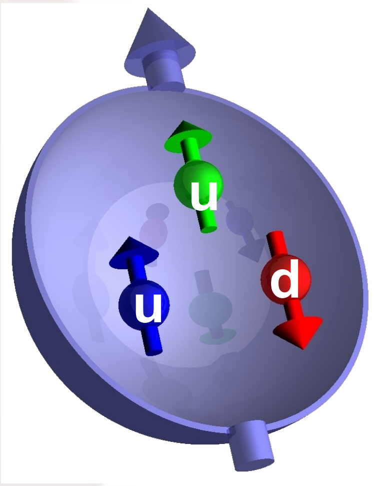
Parton distribution when nucleon is polarized?



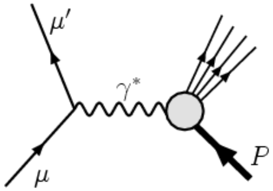
Proton Spin

- The story of the proton spin began with the quark model in 60's
- In the simple Quark Model, the nucleon is made of three quarks (nothing else)
- Because all the quarks are in the s-orbital, its spin ($\frac{1}{2}$) should be carried by the three quarks
- European Muon Collaboration: 1988

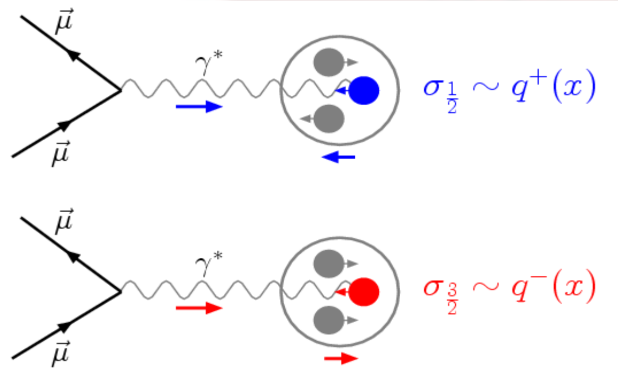
  "Spin Crisis"--- proton spin carried by quark spin is rather small

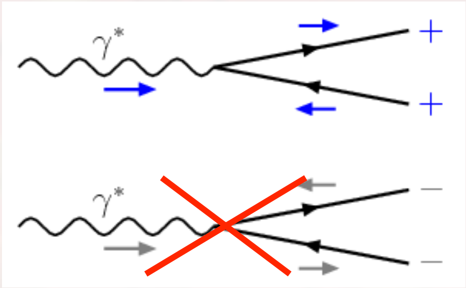


EMC experiment at CERN



- Polarized muon + p deep inelastic scattering,



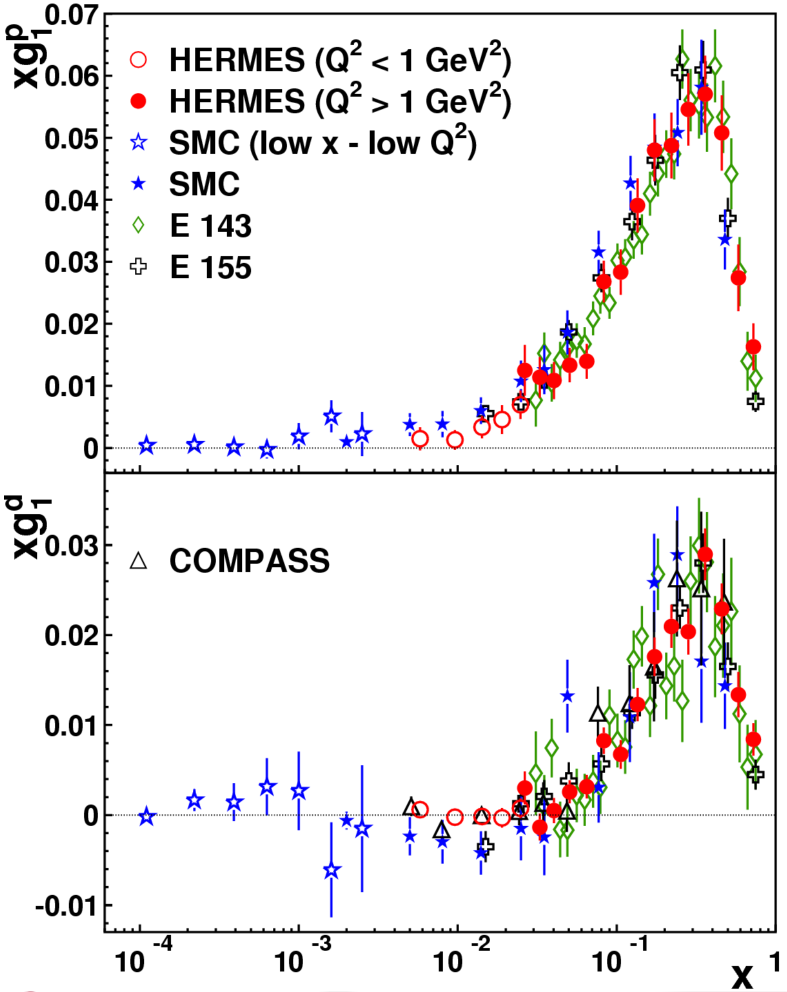


  - Virtual photon can only couple to quarks with opposite spin, because of angular momentum conservation
  - Select $q^+(x)$ or $q^-(x)$ by changing the spin direction of the nucleon or the incident lepton
  - The polarized structure function measures the quark spin density

$$g_1(x) \sim \left(\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}}\right) \propto \sum_q e_q^2 \left(\underbrace{q^+(x) - q^-(x)}\right)$$

$$\Delta q(x)$$

Summary of the polarized DIS data



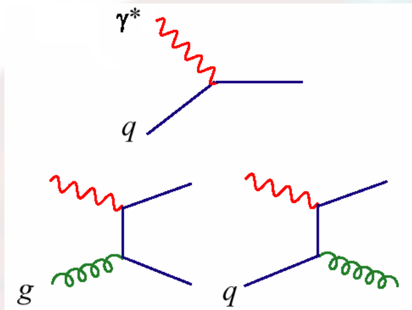
- The follow-up experiments confirm the EMC results
  - SLAC: E142-155
  - HERMES
  - SMC
  - COMPASS
- The combination of the polarized structure functions from proton and neutron leads to the total quark helicity contribution
- Quarks only carry 30% of the proton spin

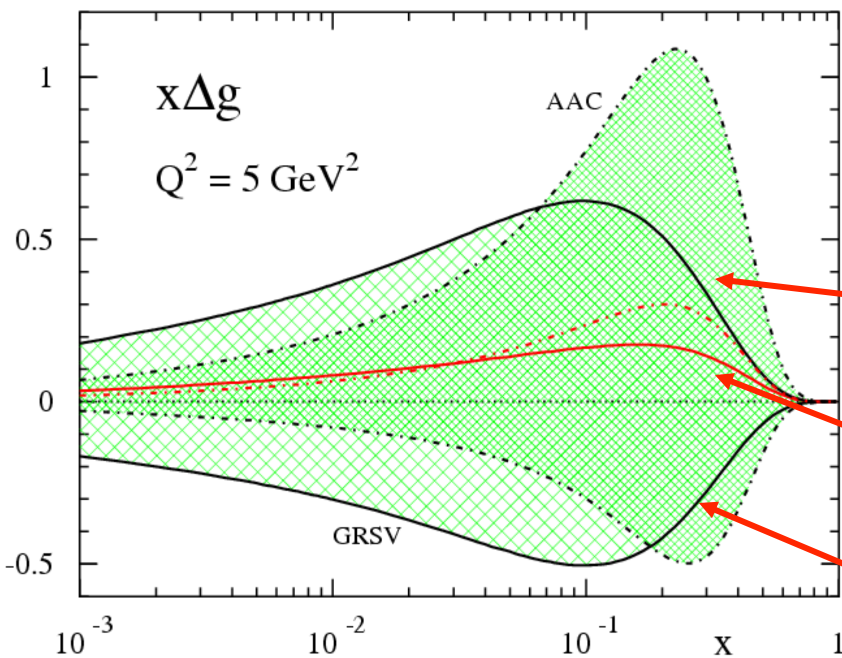
The gluon spin distribution

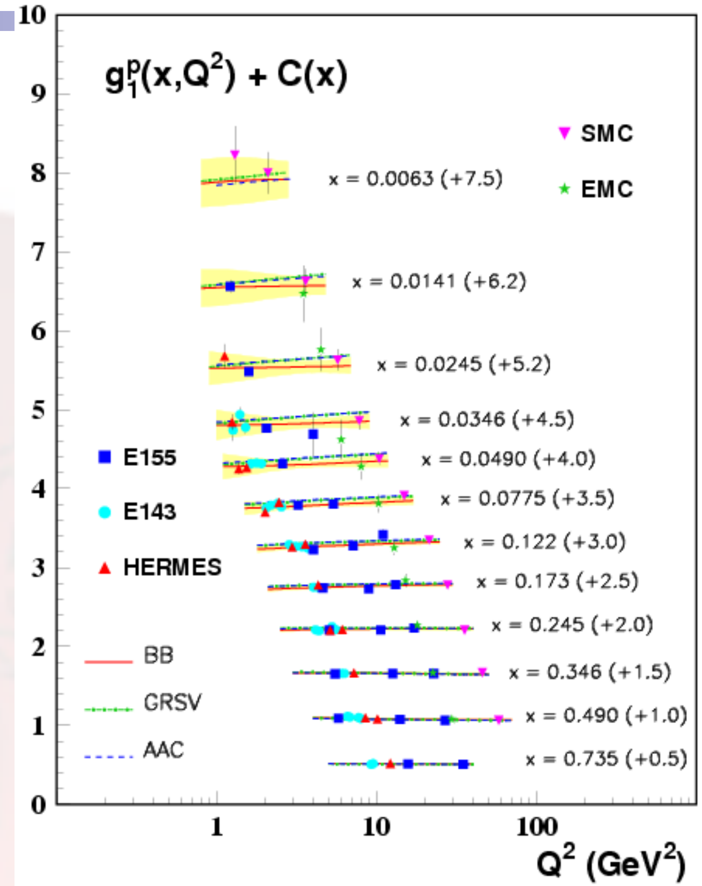
Inclusive DIS: gluon is sub-dominant

LO

NLO







$\Delta G \approx 1.8 (@1 GeV^2)$

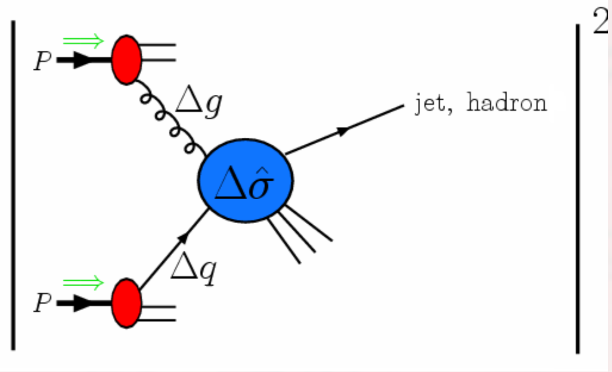
"axial anomaly", Altarelli, et al.

$\Delta G \approx 0.4$

$\Delta G \approx -1.7$

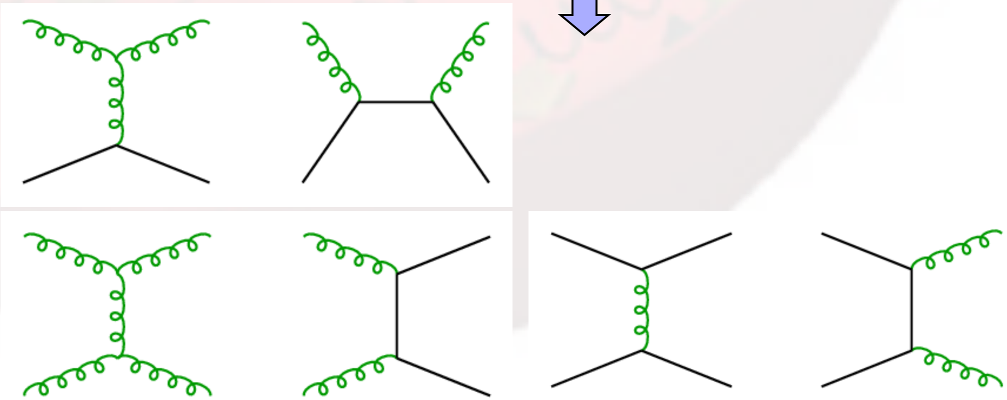
Measurement of Δg a major emphasis at RHIC

In hadronic reactions, gluons are "leaders".

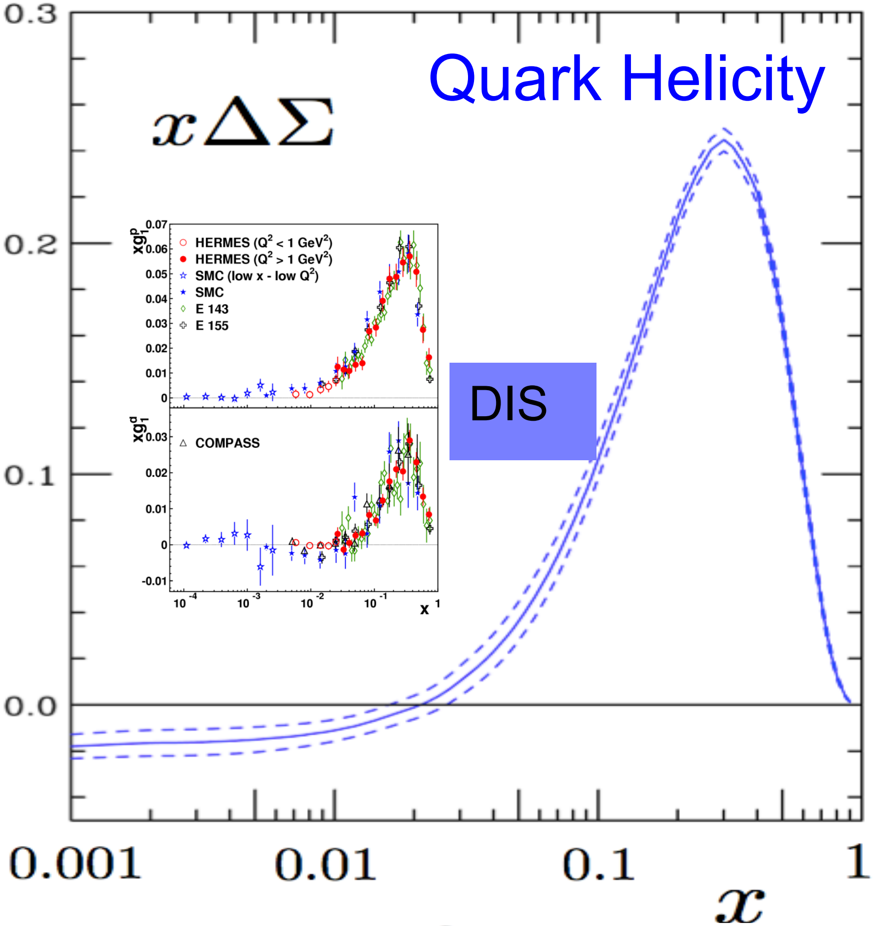
$\Delta\sigma \sim$ 

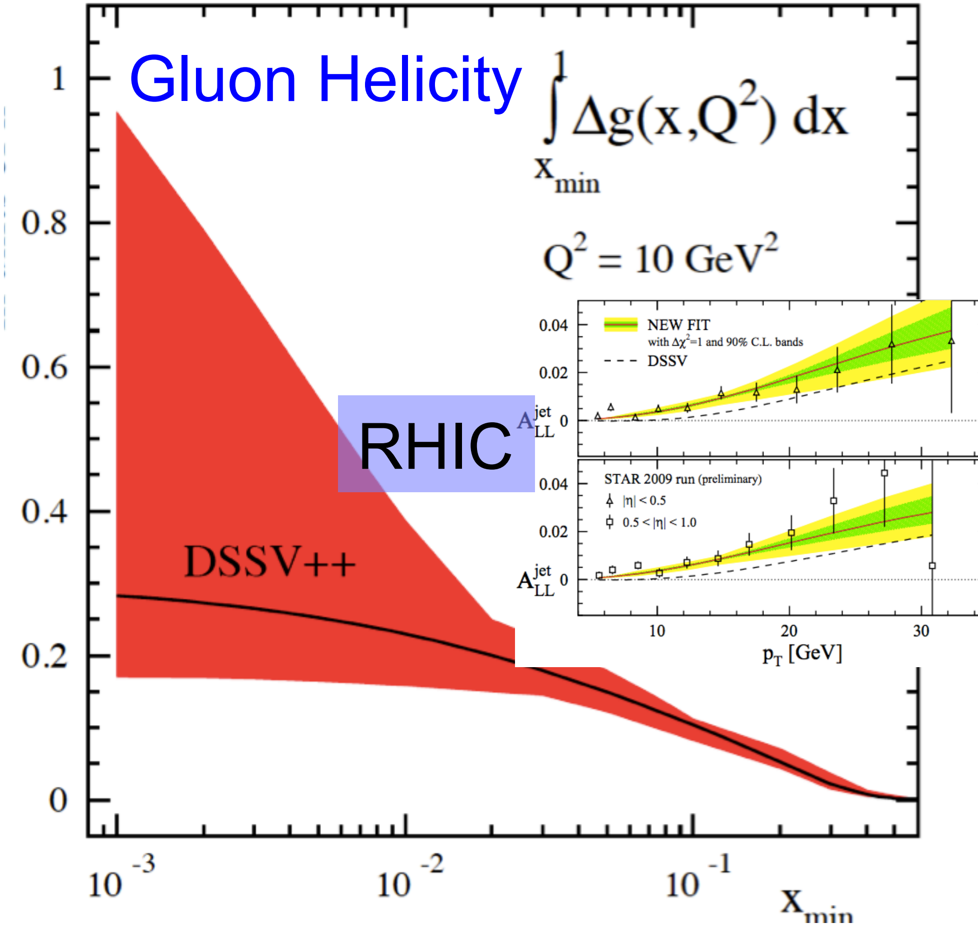
$$\Delta\sigma = \int \frac{dx_1 dx_2}{x_1 x_2} \Delta g(x_1) \Delta q(x_2) \left[\Delta\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi} \Delta\hat{\sigma}^{(1)} + \cdots \right]$$

LO



Parton distributions in a polarized nucleon





de Florian-Sassot-Stratmann-Vogelsang, 2014

Proton spin: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$

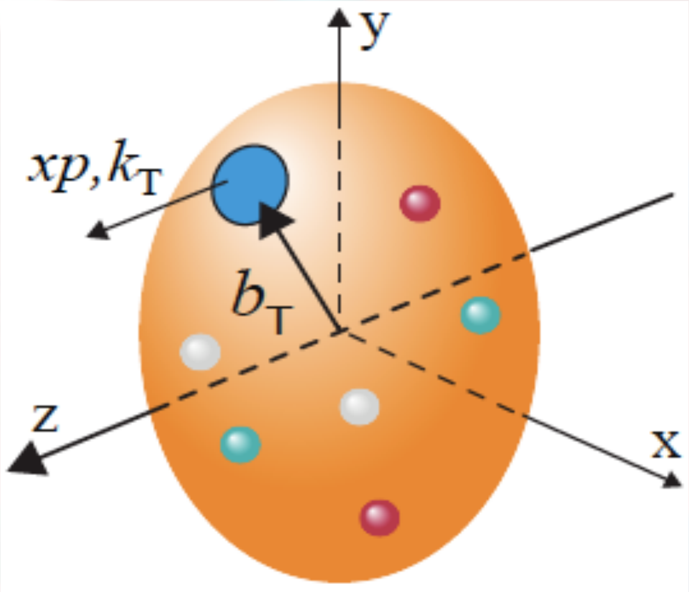
emerging phenomena?

- We know fairly well how much quark helicity contributions, $\Delta\Sigma=0.3\pm0.05$
  - Start to constrain the sea polarization (SIDIS@HERMES/COMPASS and W@RHIC)
  - Large-x and small-x? (JLab12,EIC)
- With large errors we know gluon helicity contribution plays an important role
- No direct information on quark and gluon orbital angular momentum contributions

The orbital motion:

- Orbital motion of quarks and gluons must be significant inside the nucleons!
- Orbital motion shall generate direct orbital Angular Momentum which must contribute to the spin of the proton
- Orbital motion can also give rise to a range of interesting physical effects (Single Spin Asymmetries)

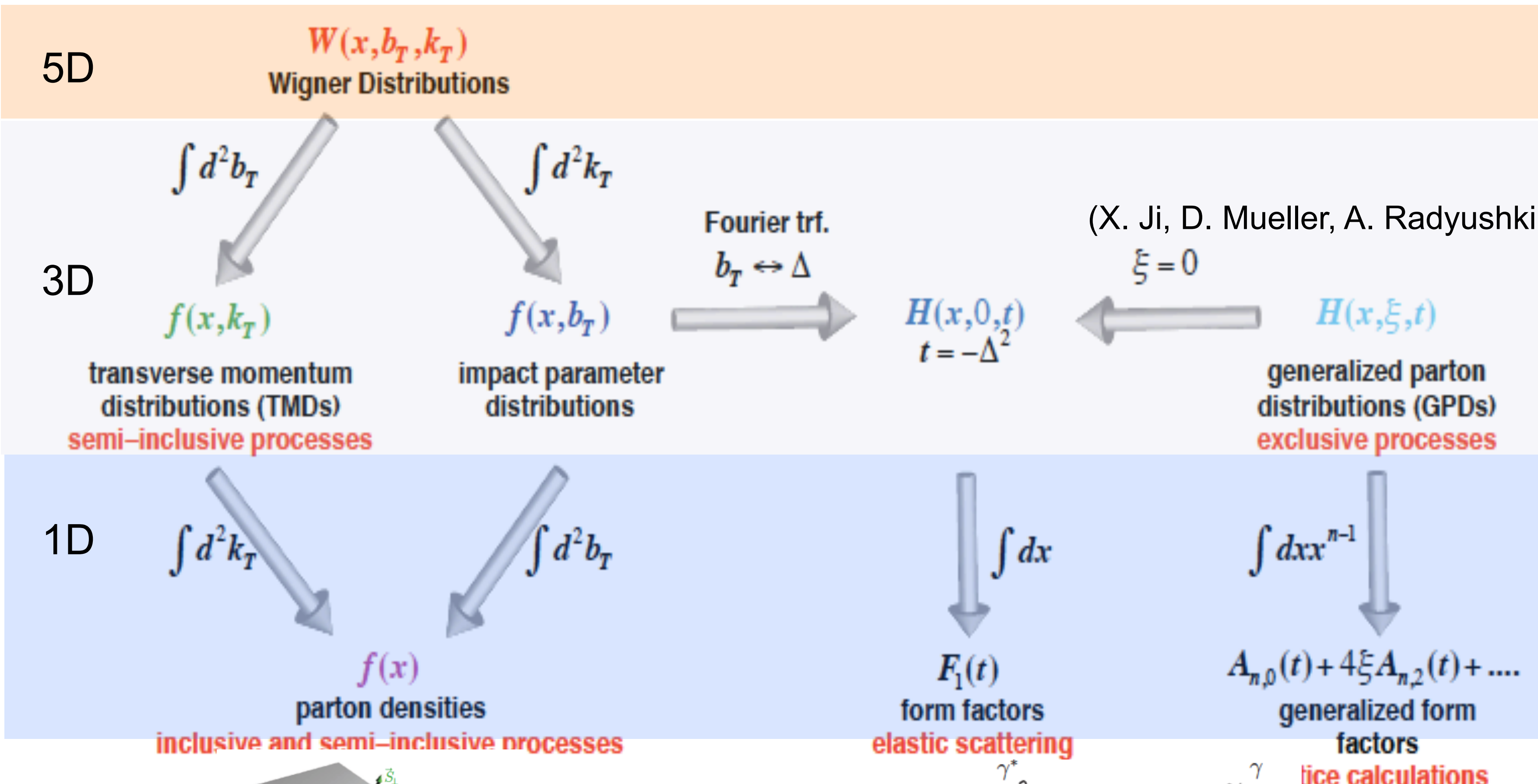
New ways to look at partons

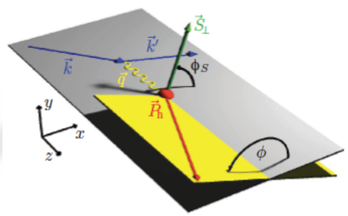


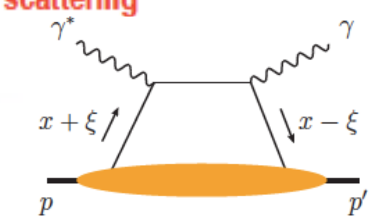
- Partons in transverse coordinate space
  - Generalized parton distributions (GPDs)
- Partons in transverse momentum space
  - Transverse-momentum distributions (TMDs)
- Both? Wigner distributions!

Unified view of the Nucleon

- **Wigner distributions (Belitsky, Ji, Yuan)**



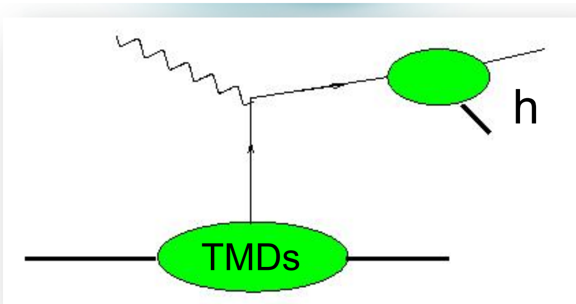


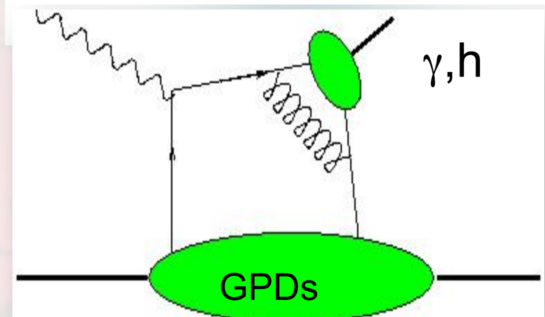


Zoo of TMDs & GPDs

	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_1, h_{1T}^{\perp}$

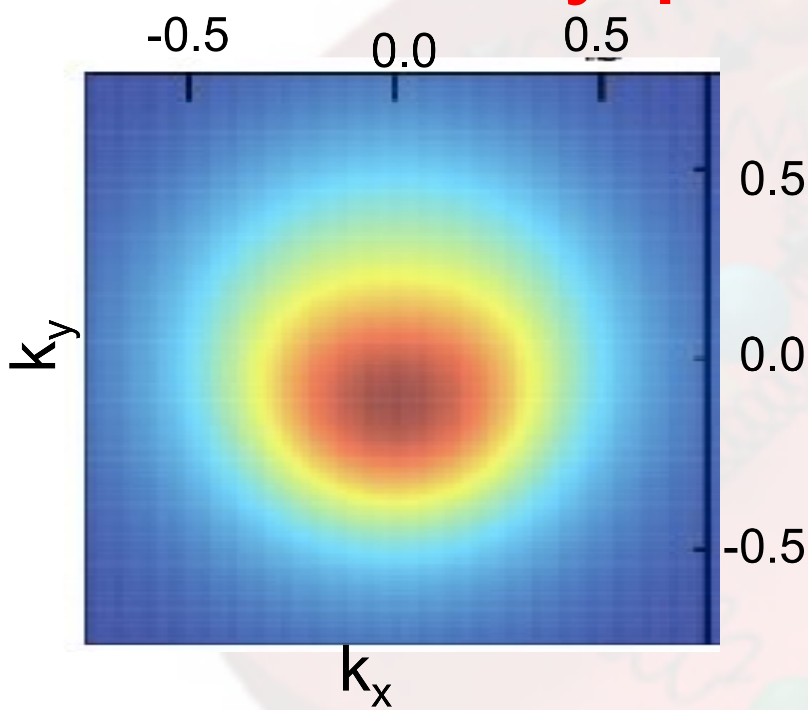
	U	L	T
U	H		$\mathcal{E}_T$
L		$\widetilde{H}$	
T	E		$H_T, \widetilde{H}_T$

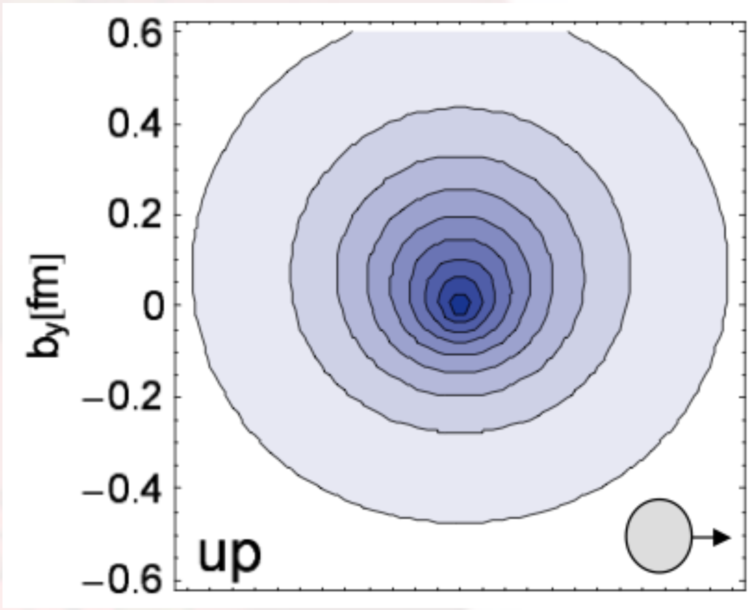




- NOT directly accessible
- Their extractions require measurements of x-sections and asymmetries in **a large kinematic domain of** x_B, t, Q^2 (GPD) and x_B, P_T, Q^2, z (TMD)

Deformation when nucleon is transversely polarized





Quark Sivers function fit to the SIDIS Data, Anselmino, et al. 2009

Lattice Calculation of the transvese density Of Up quark, QCDSF/UKQCD Coll., 2006

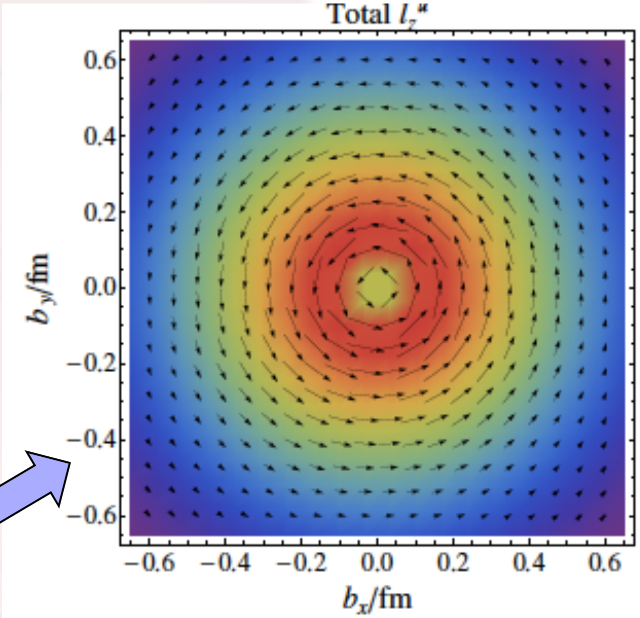
Parton's orbital motion through the Wigner Distributions

Phase space distribution:

Projection onto p (x) to get the momentum (probability) density

Quark orbital angular momentum

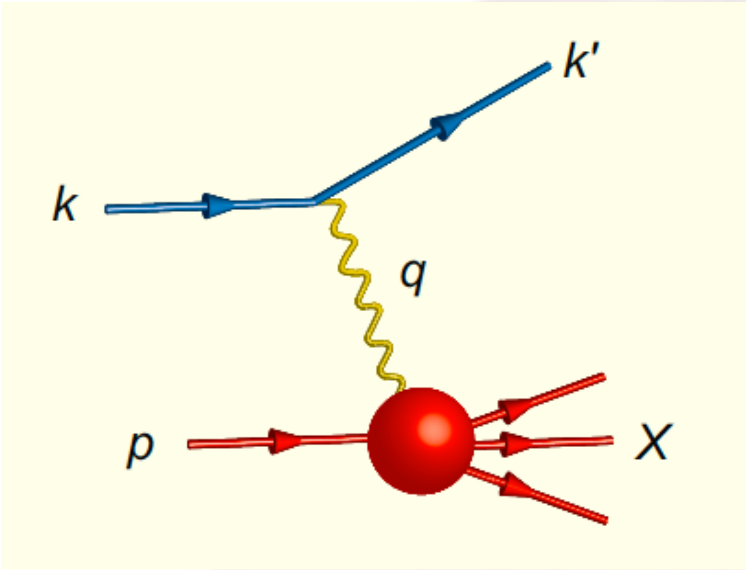
$$L(x) = \int (\vec{b}_\perp \times \vec{k}_\perp) W(x, \vec{b}_\perp, \vec{k}_\perp) d^2\vec{b}_\perp d^2\vec{k}_\perp$$



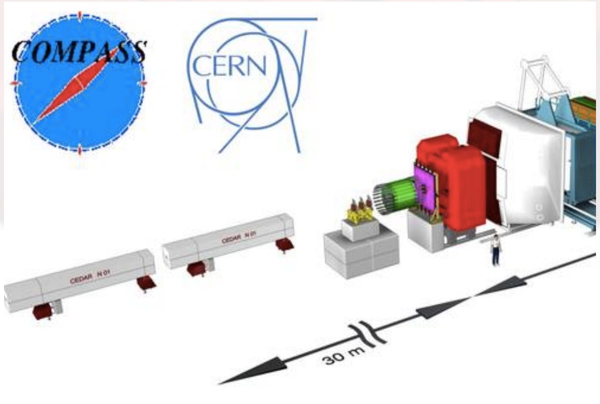
Well defined in QCD:

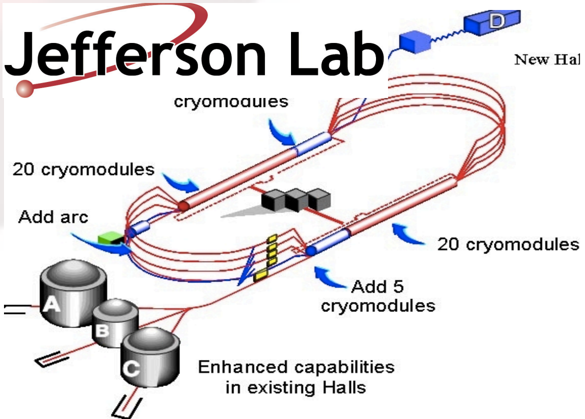
Ji, Xiong, Yuan, PRL, 2012; PRD, 2013
Lorce, Pasquini, Xiong, Yuan, PRD, 2012

Where can we study: Deep Inelastic Scattering



- Inclusive DIS
  - Parton distributions
- Semi-inclusive DIS, measure additional hadron in final state
  - Kt-dependence
- Exclusive Processes, measure recoiled nucleon
  - Nucleon tomography





What we have learned

- Unpolarized transverse momentum (coordinate space) distributions from, mainly, DIS, Drell-Yan, W/Z boson productions, (HERA exp.)
- Indications of polarized quark distributions from low energy DIS experiments (HERMES, COMPASS, JLab)

What we are missing

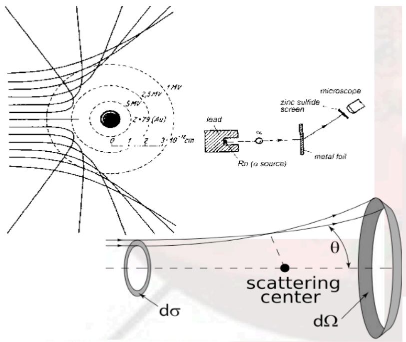
- Precise, detailed, mapping of polarized quark/gluon distribution
  - Universality/evolution more evident
- Spin correlation in momentum and coordinate space/tomography
  - Crucial for orbital motion
- Small-x: links to other hot fields (Color-Glass-Condensate)

Perspectives

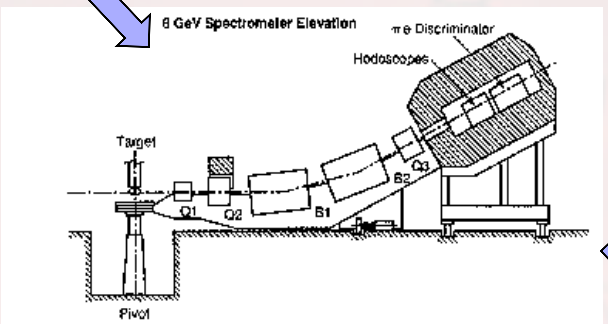
- HERA (ep collider) limited by the statistics, and not polarized in both beams
- Existing fixed target experiments are limited by statistics and kinematics
- JLab 12 will provide un-precedent data with high luminosity
- Ultimate machine will be the Electron-Ion-Collider (EIC): kinematic coverage with high luminosity

Recap of yesterday

Landscape of Atomic Matter



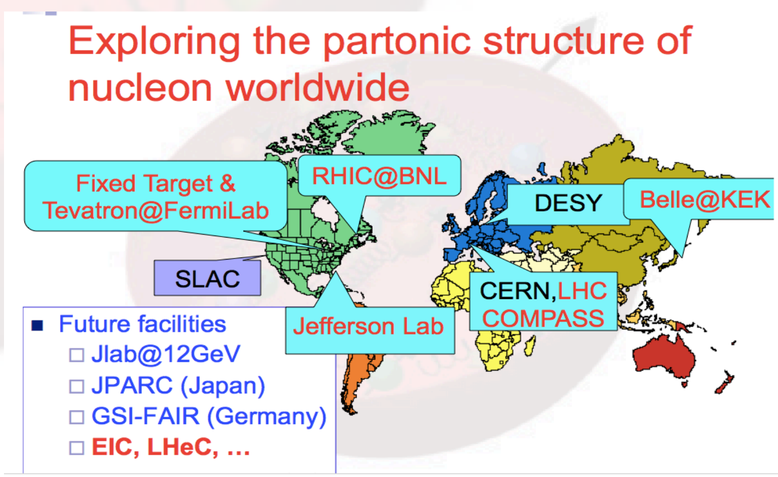
Rutherford Scattering, 1911
- Discovery of nucleus



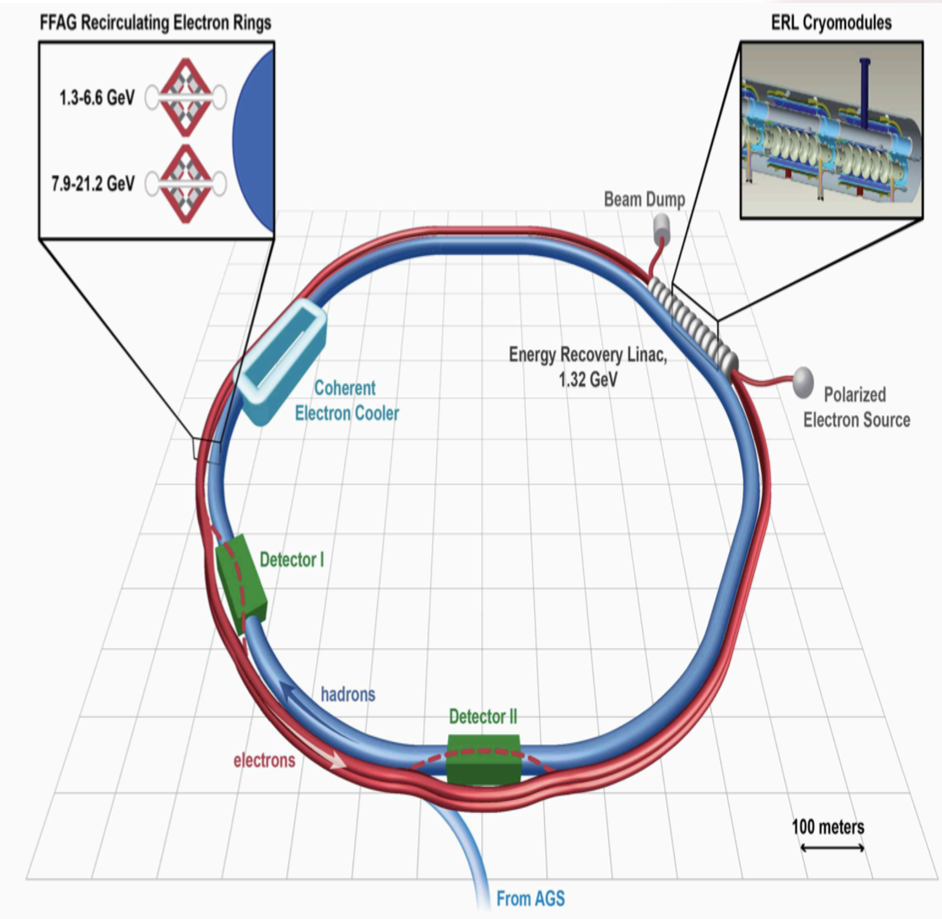
DIS at SLAC, 1960s
- Discovery of quarks

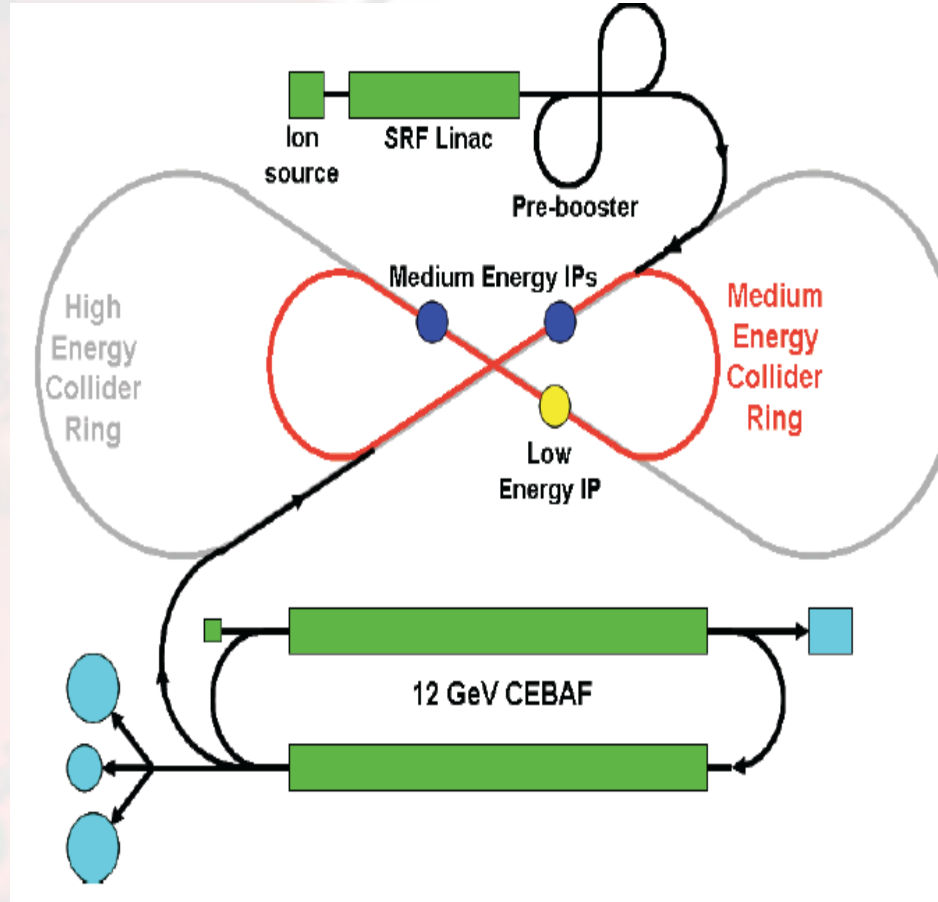
Quantum ChromoDynamics:

$$L = \bar{\psi}(i\gamma \cdot \partial - m_q)\psi - \frac{1}{4}F^{\mu\nu a}F_{\mu\nu a} - g_s\bar{\psi}\gamma \cdot A\psi$$



EIC Proposals in US

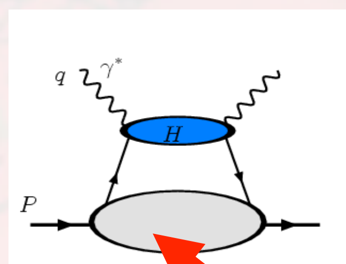


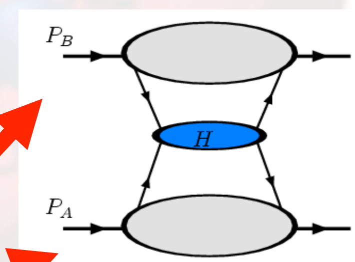


arXiv: 1108.1713, arXiv: 1212.1701

Feynman's parton language and QCD Factorization

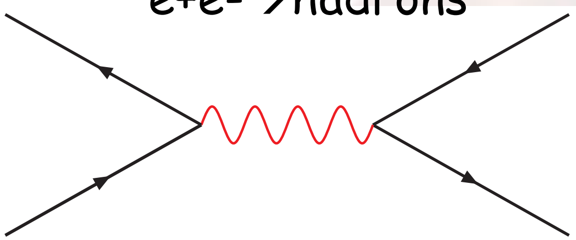
- In high-energy hadronic reactions, the scattering can be decomposed into a convolution of parton scattering and parton density (distribution), or wave function or correlations
  - QCD Factorization!



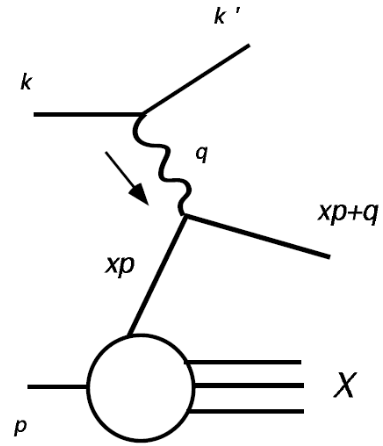


$$\sim \int \text{Parton Distributions} \otimes \text{Hard Partonic Cross Section}$$

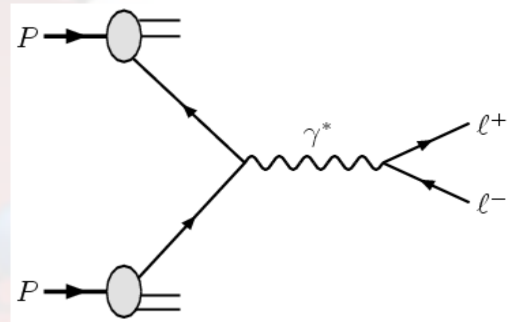
e+e-→hadrons



Deep Inelastic Scattering



Drell-Yan Process

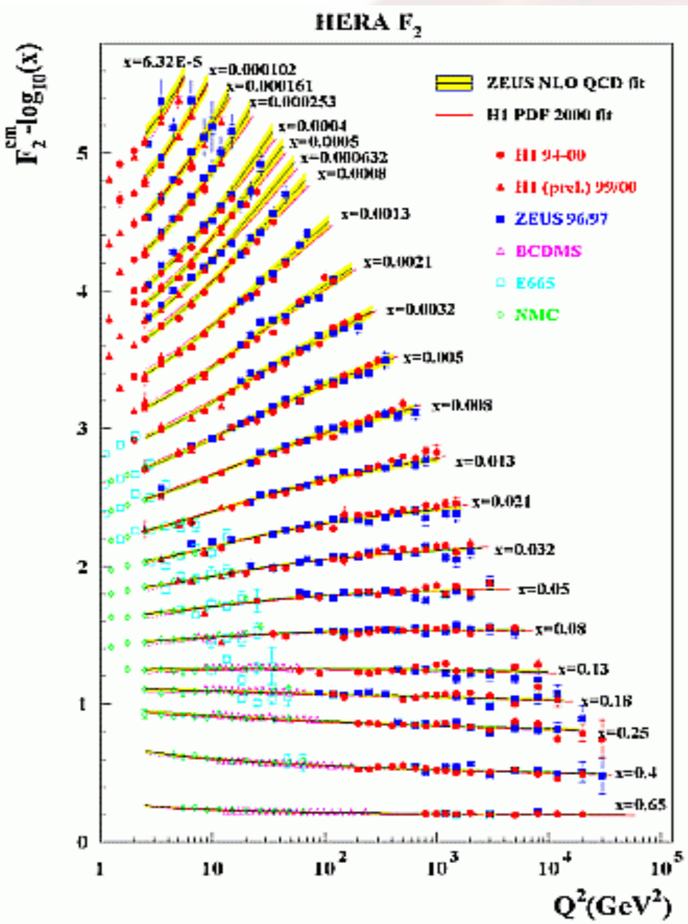


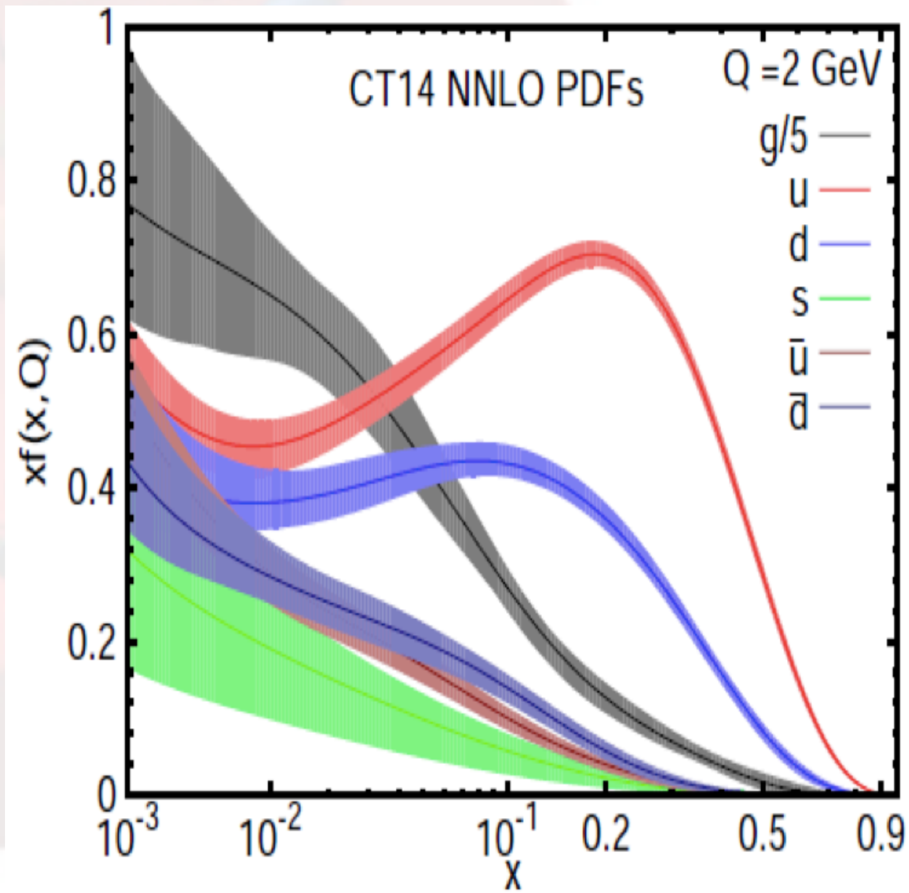
$$\sigma(e^+e^- \to q\bar{q}) = N_c \frac{4\pi}{3}\frac{\alpha^2}{Q^2}e_q^2$$

- Universal parton distributions between DIS and Drell-Yan Processes
- Partonic cross sections can be calculated perturbatively

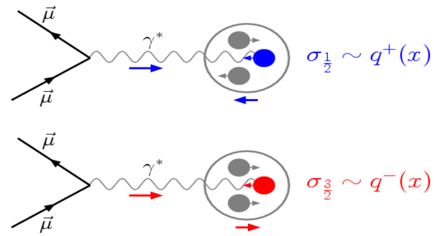
QCD dynamics

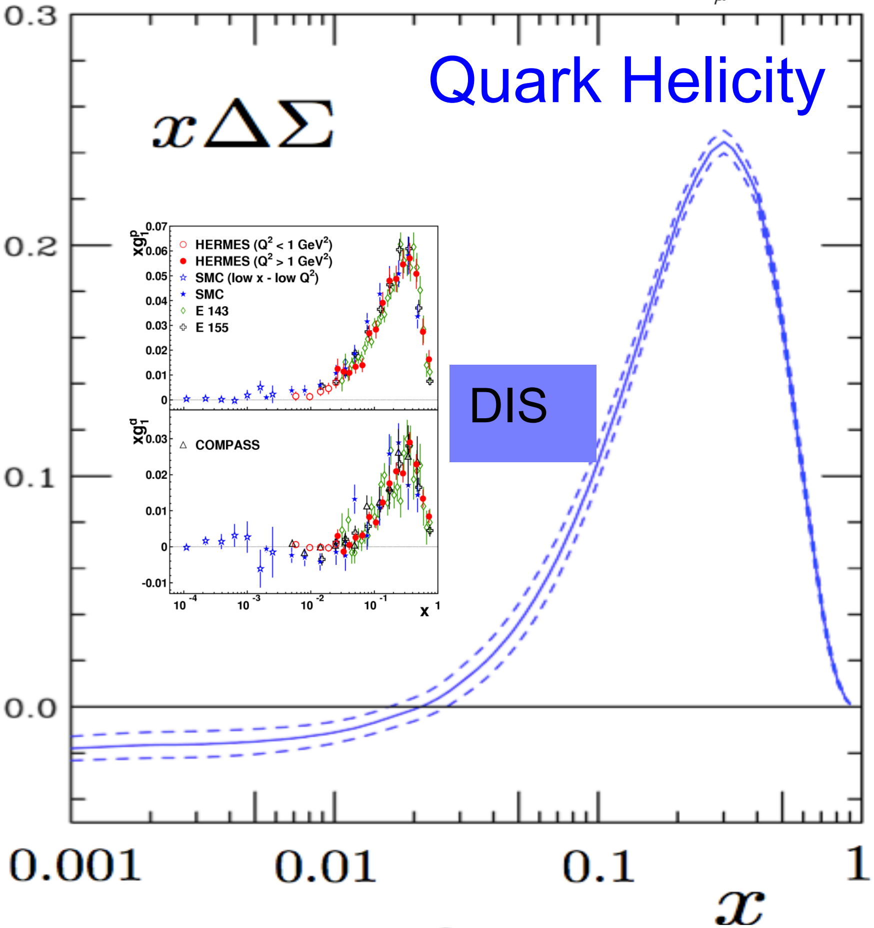
$$\mu\frac{d^2}{d\mu^2}\phi_{i/h}(x,\mu^2) = \sum_{j=f,\bar{f},G}\int_x^1 \frac{d\xi}{\xi}P_{ij}(\frac{x}{\xi},\alpha_s(\mu^2))\phi_{j/h}(\xi,\mu^2)$$

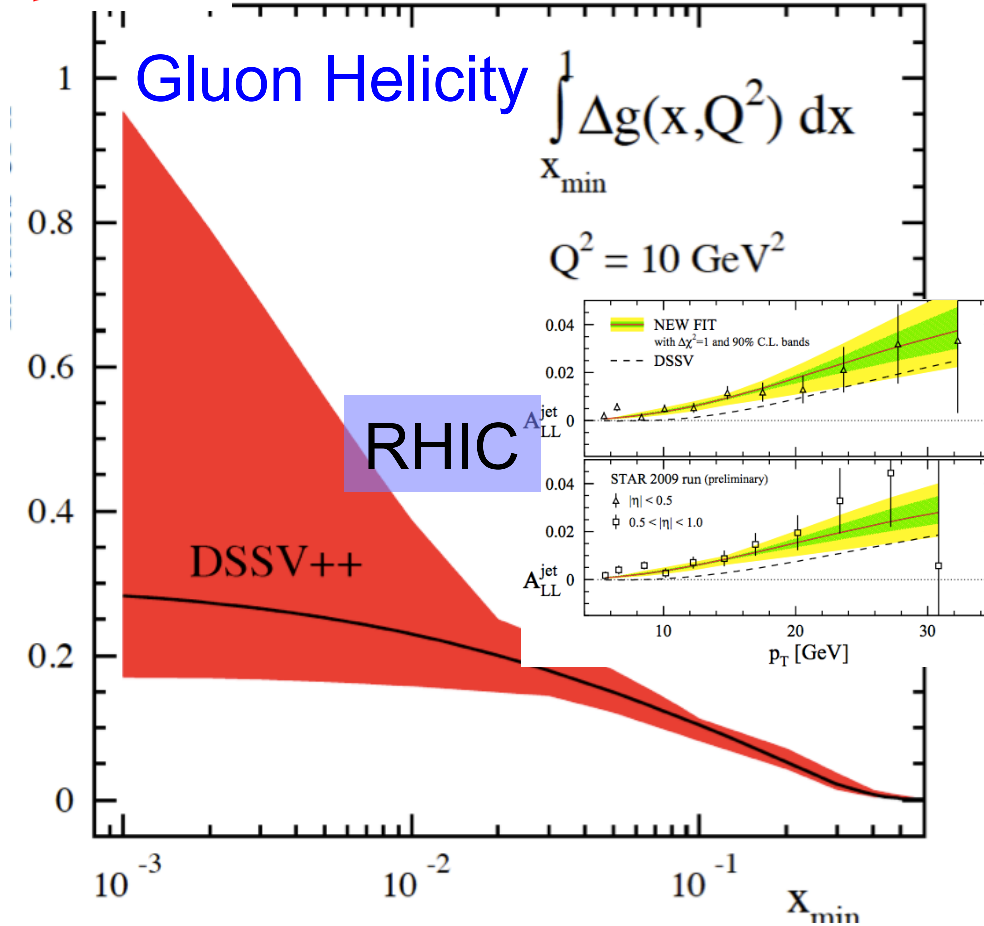




Parton distributions in a polarized nucleon







de Florian-Sassot-Stratmann-Vogelsang, 2014

BERKELEY LAB

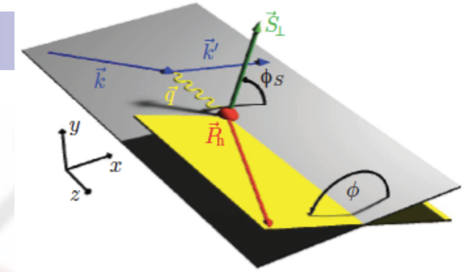
Proton spin: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$ emerging phenomena?

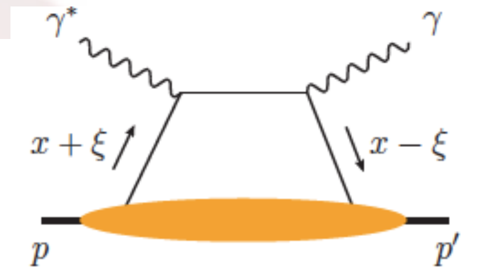
- We know fairly well how much quark helicity contributions, $\Delta\Sigma=0.3\pm0.05$
- With large errors we know gluon helicity contribution plays an important role
- No direct information on quark and gluon orbital angular momentum contributions

Extension to transverse direction…

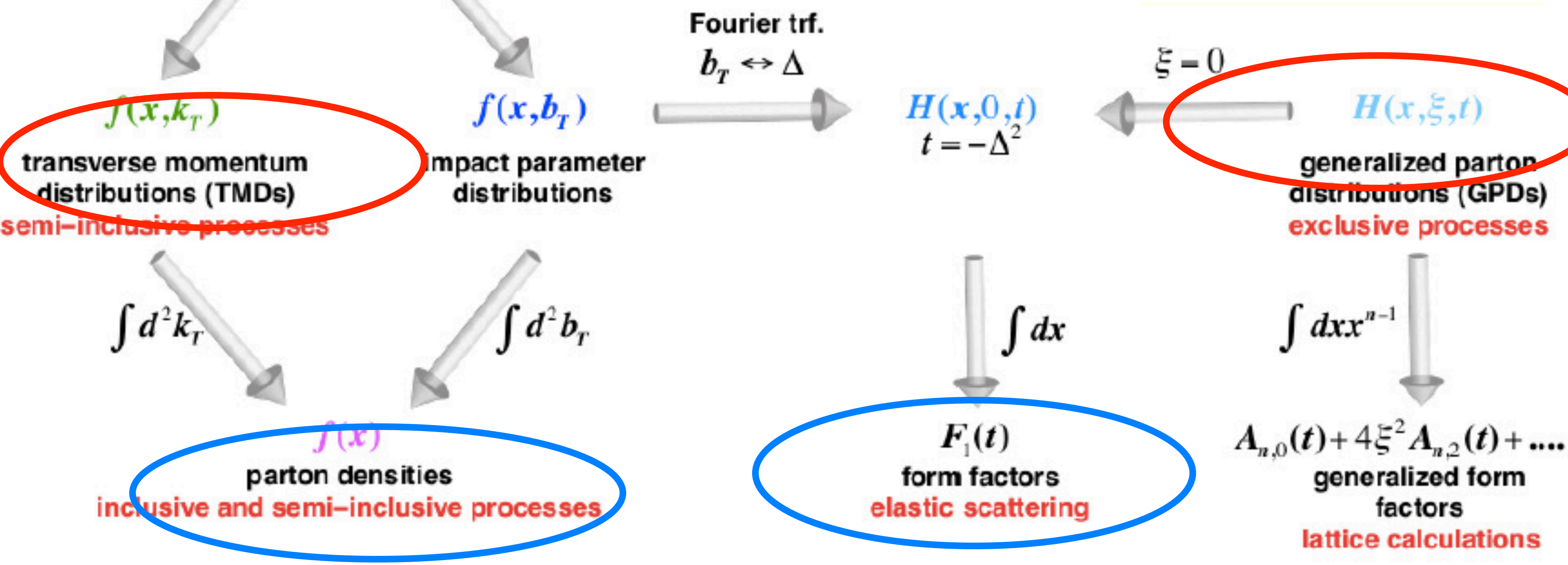
- Semi-inclusive measurements (in DIS or Drell-Yan processes)
  - Transverse momentum distributions (TMD)
- Deeply Virtual Compton Scattering and Exclusive processes
  - Generalized parton distributions (GPD)

Nucleon tomography

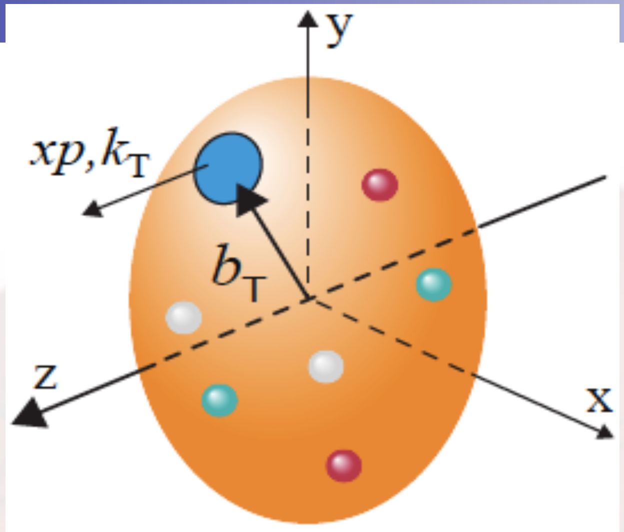




EIC Focus

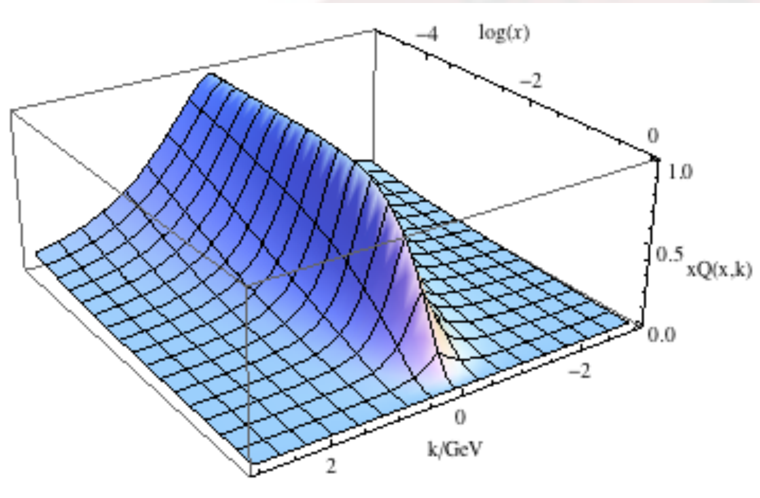


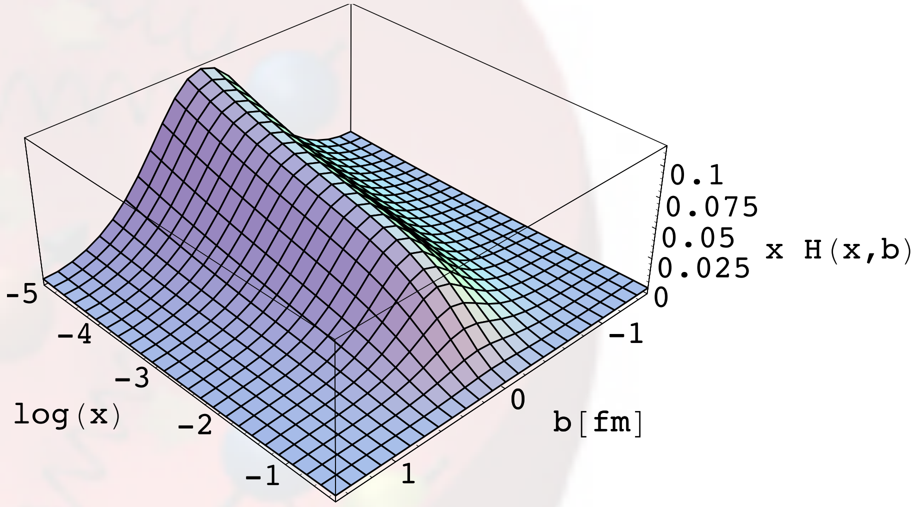
Current Lattice Simulations



- 3D Imaging from the GPDs and TMDs measurements
  - Try to answer more detailed questions as Rutherford was doing 100 years ago
- QCD dynamics involved in these processes
  - In particular for the TMD part: universality, factorization, evolutions,…

Transverse profile for the quark distribution: k_t vs b_t

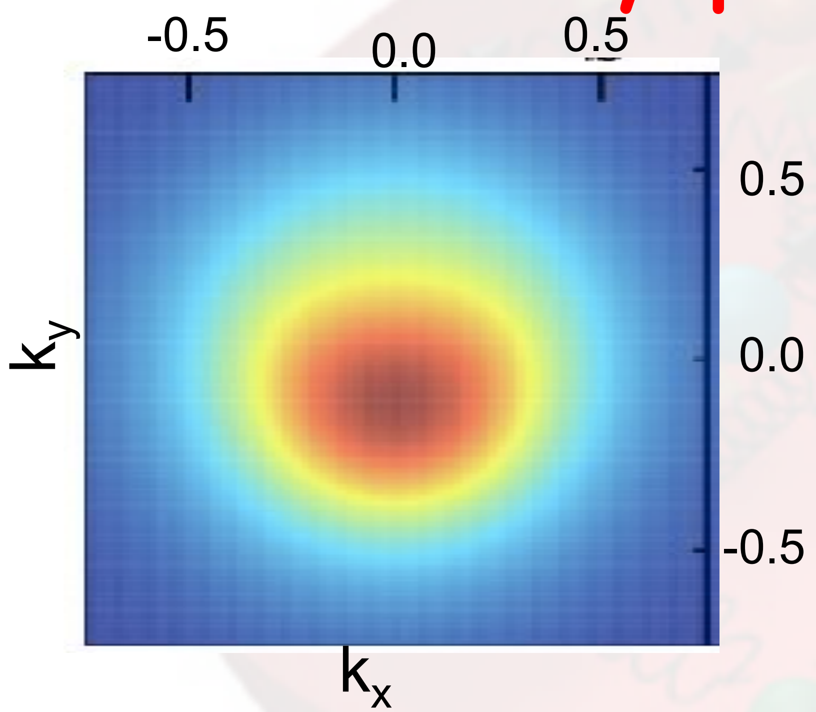




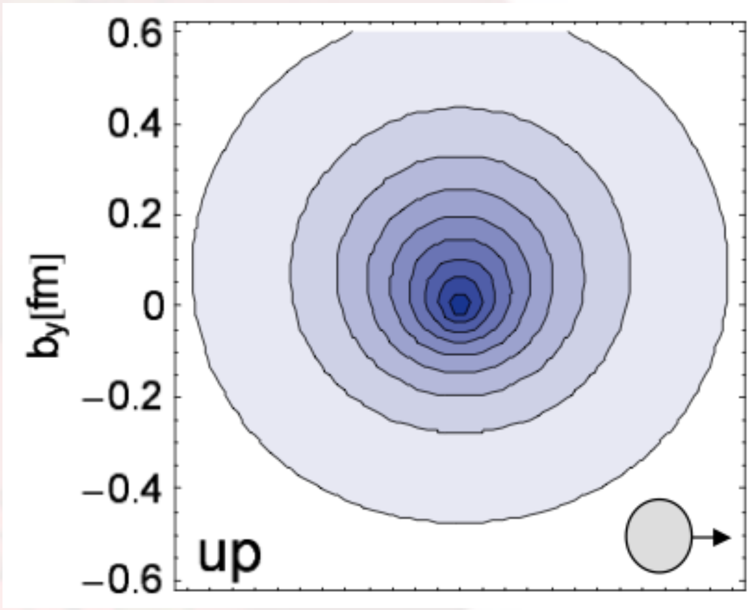
Quark distribution calculated from a saturation-inspired model
A.Mueller 99, McLerran-Venugopalan 99

GPD fit to the DVCS data from HERA, Kumerick-D.Mueller, 09,10

Deformation when nucleon is transversely polarized



Quark Sivers function fit to the SIDIS Data, Anselmino, et al. 20009



Lattice Calculation of the IP density of Up quark, QCDSF/UKQCD Coll., 2006

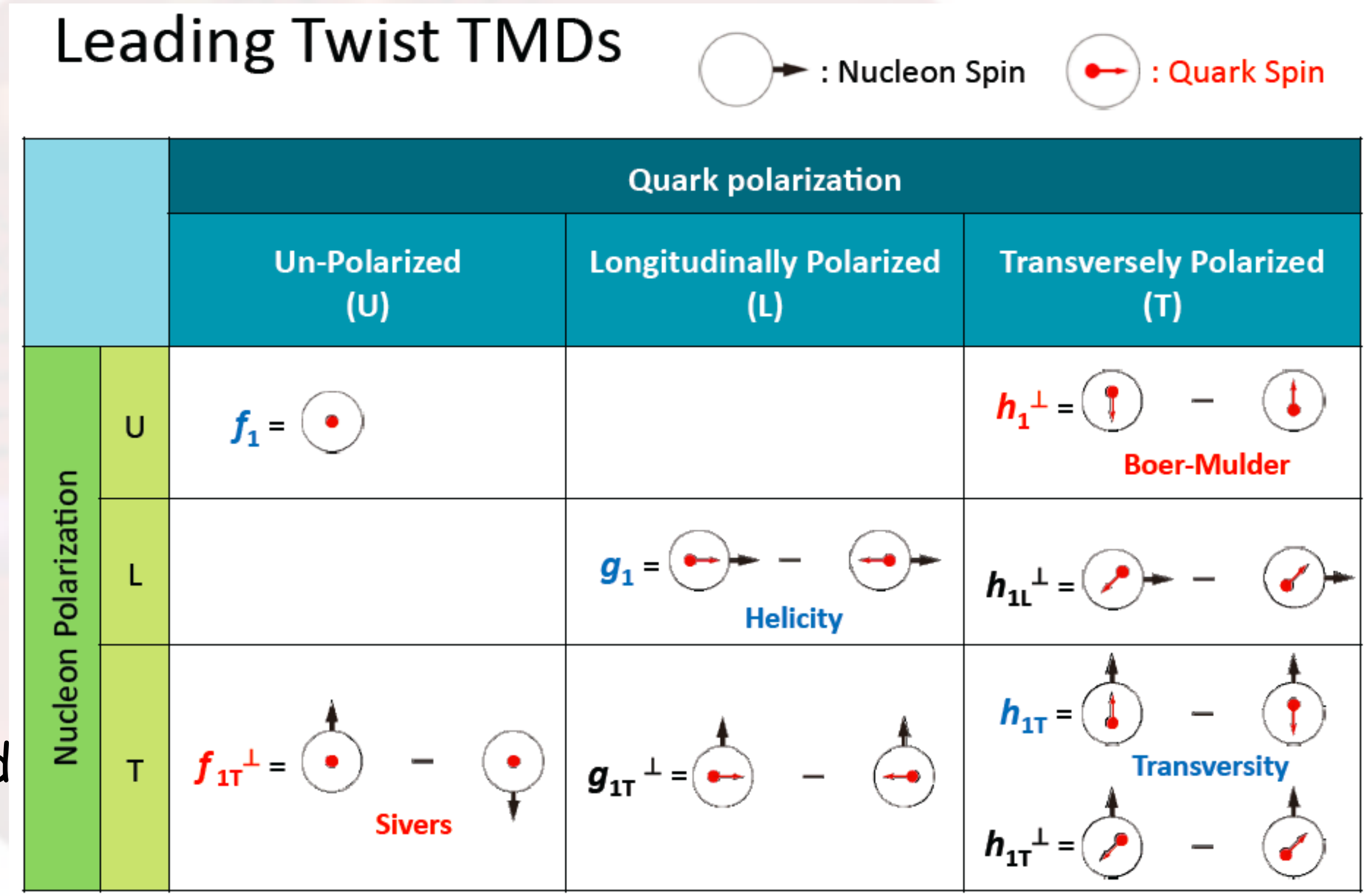
Transverse momentum distribution

Straightforward extension

- Spin average, helicity, and transversity distributions

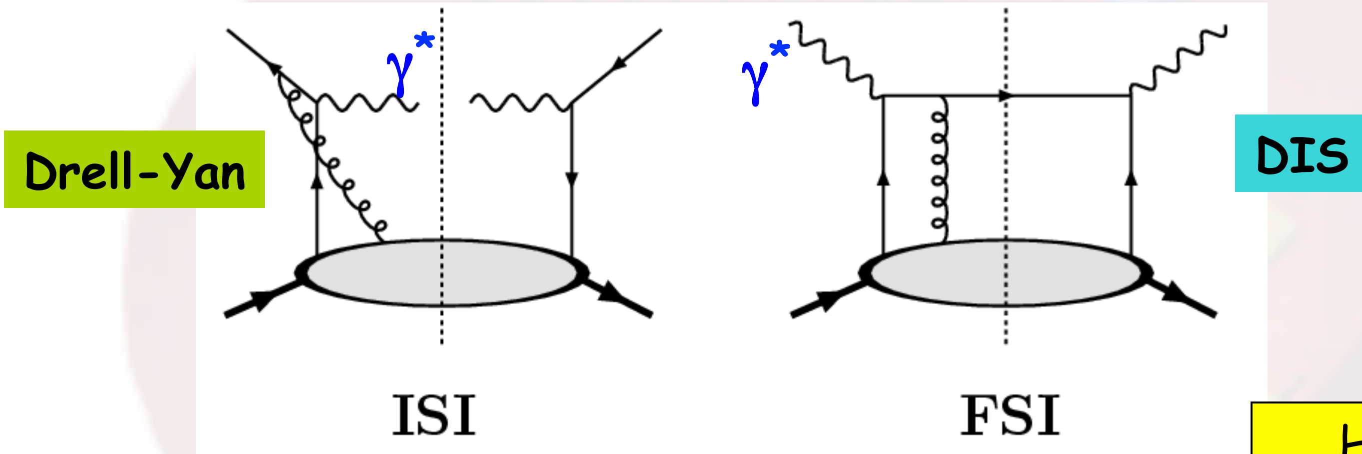
P_T-spin correlations

- Nontrivial distributions, $S_T X P_T$
- In quark model, depends on S- and P-wave interference



Sivers Asymmetries in DIS and Drell-Yan

- Initial state vs. final state interactions



$$\mathrm{Sivers}|_{DY} = -\mathrm{Sivers}|_{DIS}$$

HERMES/ COMPASS

- "Universality": QCD prediction

What we have learned

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- Indications of polarized quark distributions from low energy DIS experiments (HERMES, COMPASS, JLab)

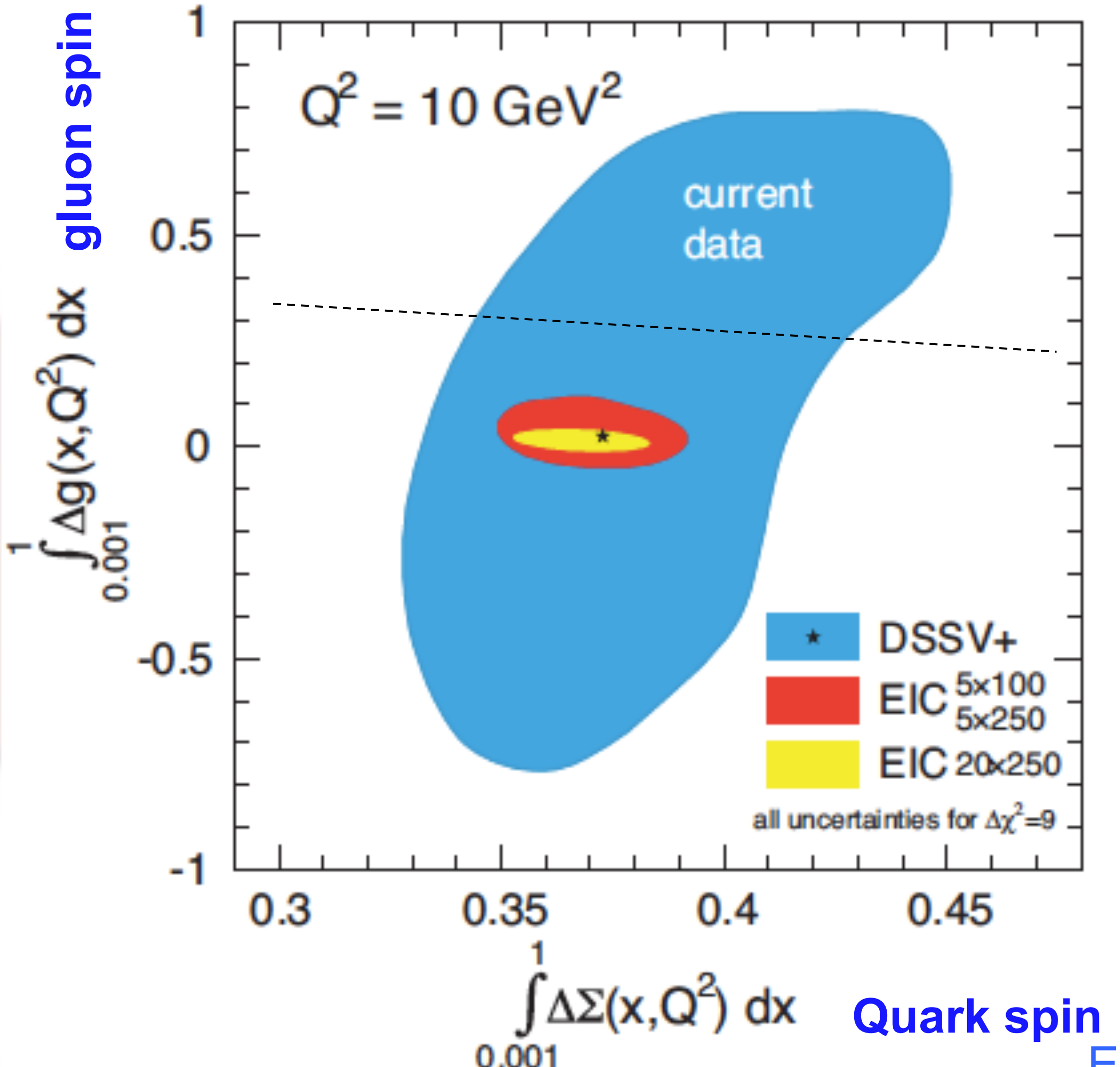
What we are missing

- Precise, detailed, mapping of polarized quark/gluon distribution
  - Universality/evolution more evident
- Spin correlation in momentum and coordinate space/tomography
  - Crucial for orbital motion
- Small-x: links to other hot fields (Color-Glass-Condensate)

EIC: Understanding the glue that bind us all

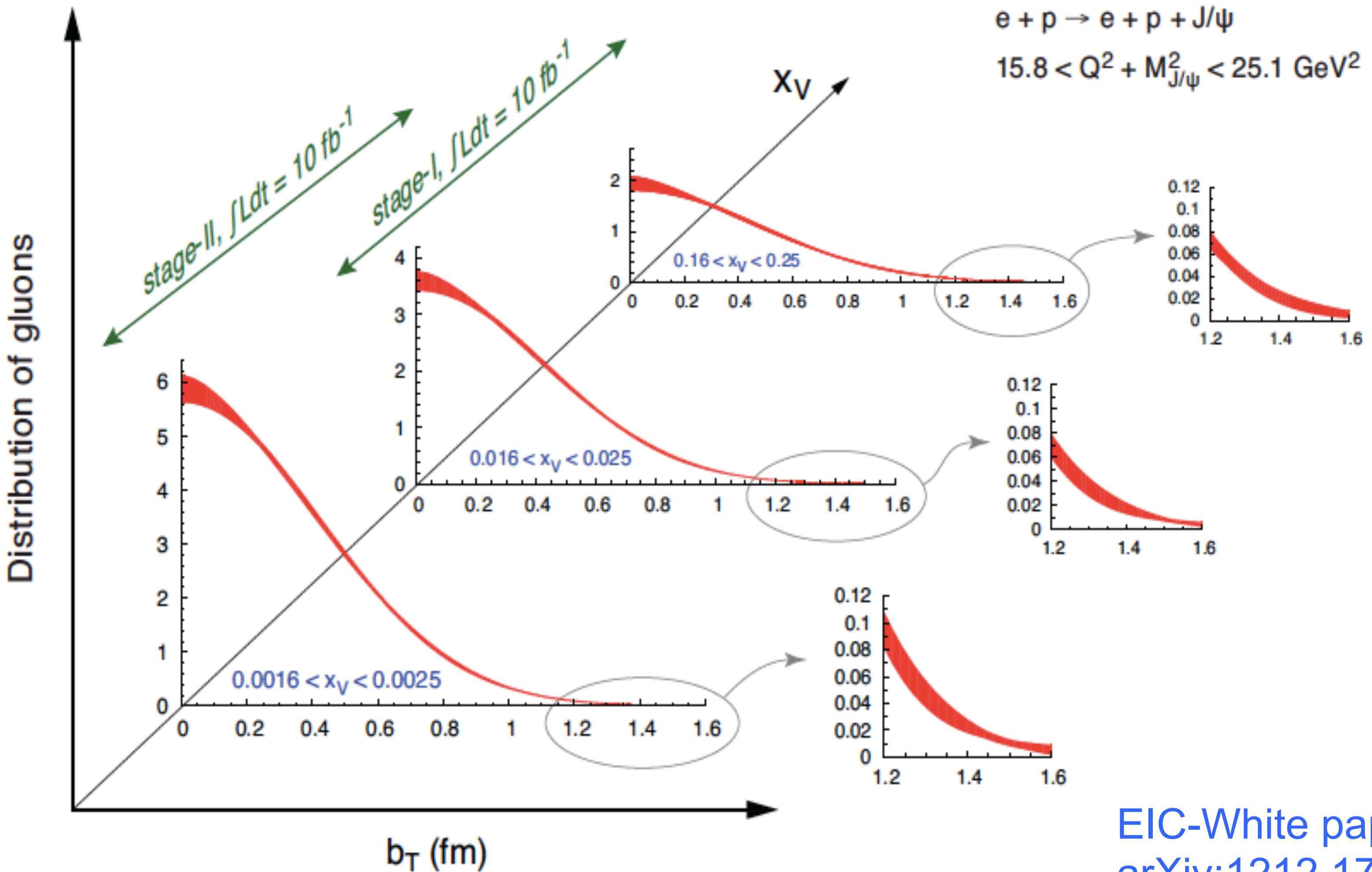
- Gluon plays an important role in the momentum of the nucleon
- Nucleon spin structure through helicity ΔG
- Gluon orbital motion
  - Nucleon tomography (orbital-spin correlations)
- Small x: gluon saturation (CGC)-> a saturated transverse-momentum distribution

Stratmann, et al.
EIC-White Paper



EIC-White paper
arXiv:1212.1701

Gluon tomography at small x (GPDs)



EIC-White paper
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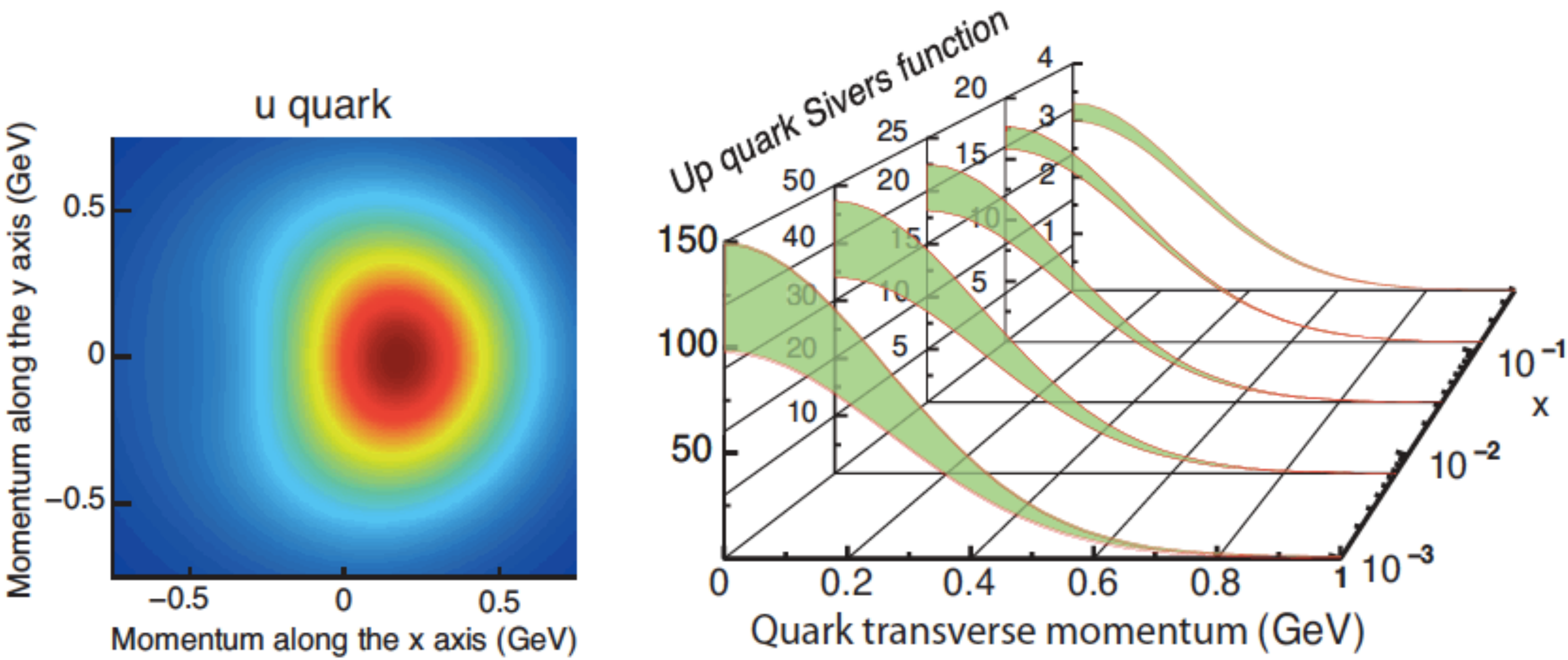
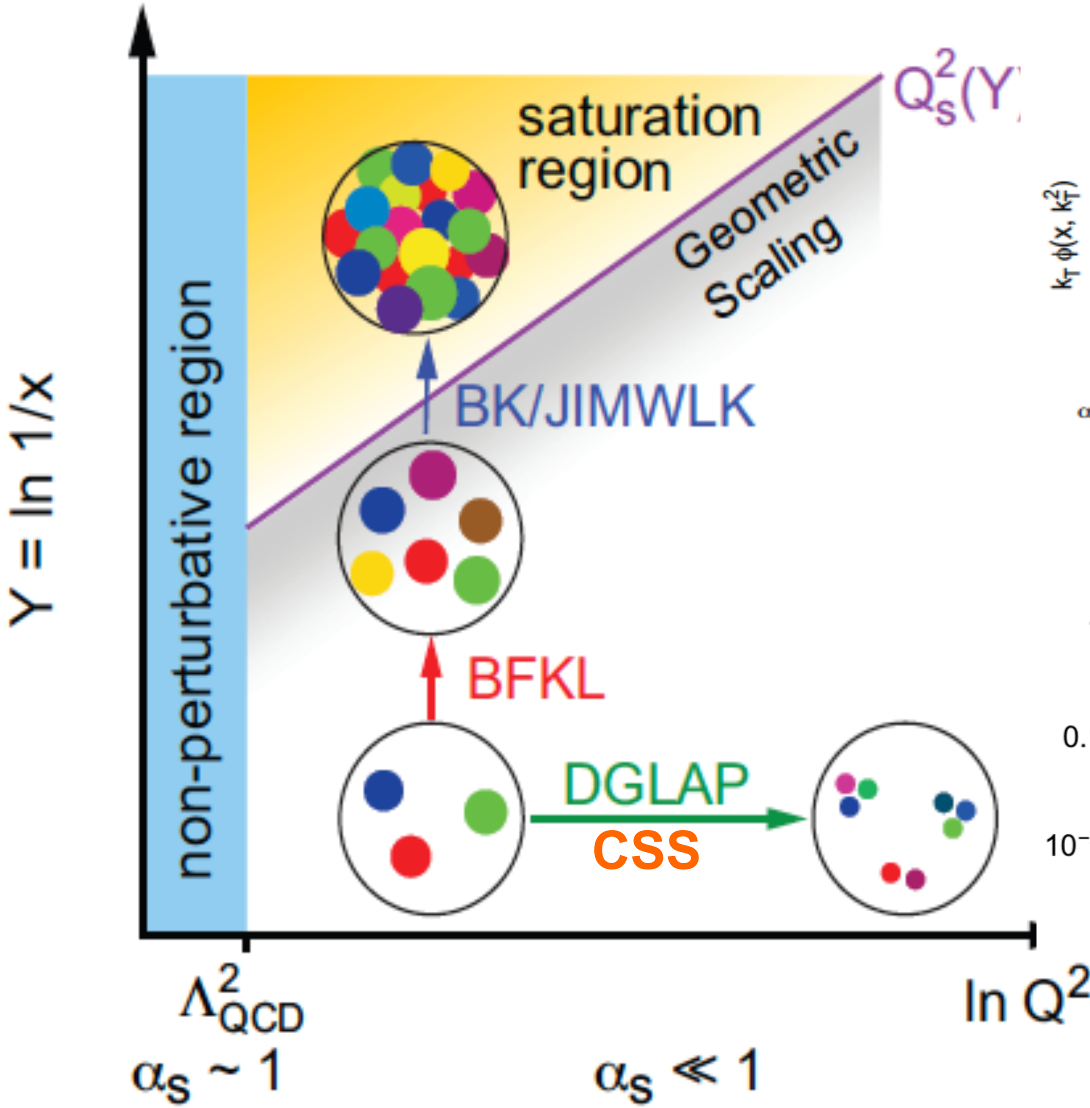
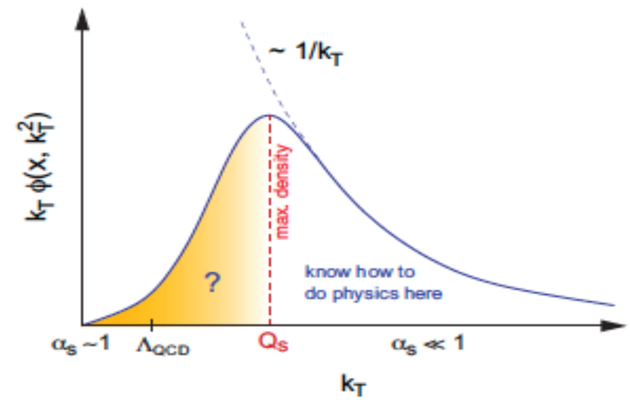
Transverse momentum distributions

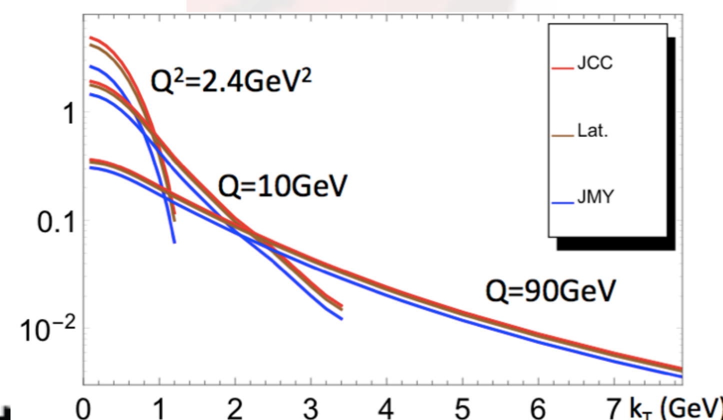


EIC-White paper
arXiv:1212.1701

Transverse momentum distributions:
A unified picture





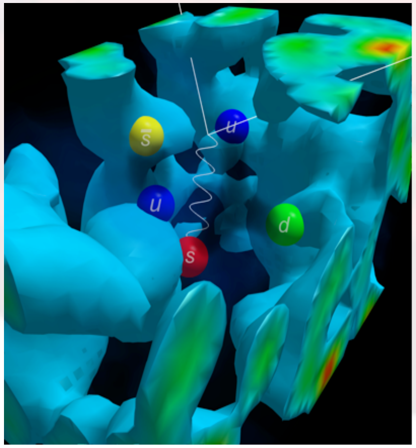


Theoretical Issues

- New structure, new dynamics and new phenomena!
  - New Structure and probe physics separation or factorization
  - New processes to measure novel observables
    - Spin correlation to study orbital motion
  - Study partons directly on lattice

Lattice QCD

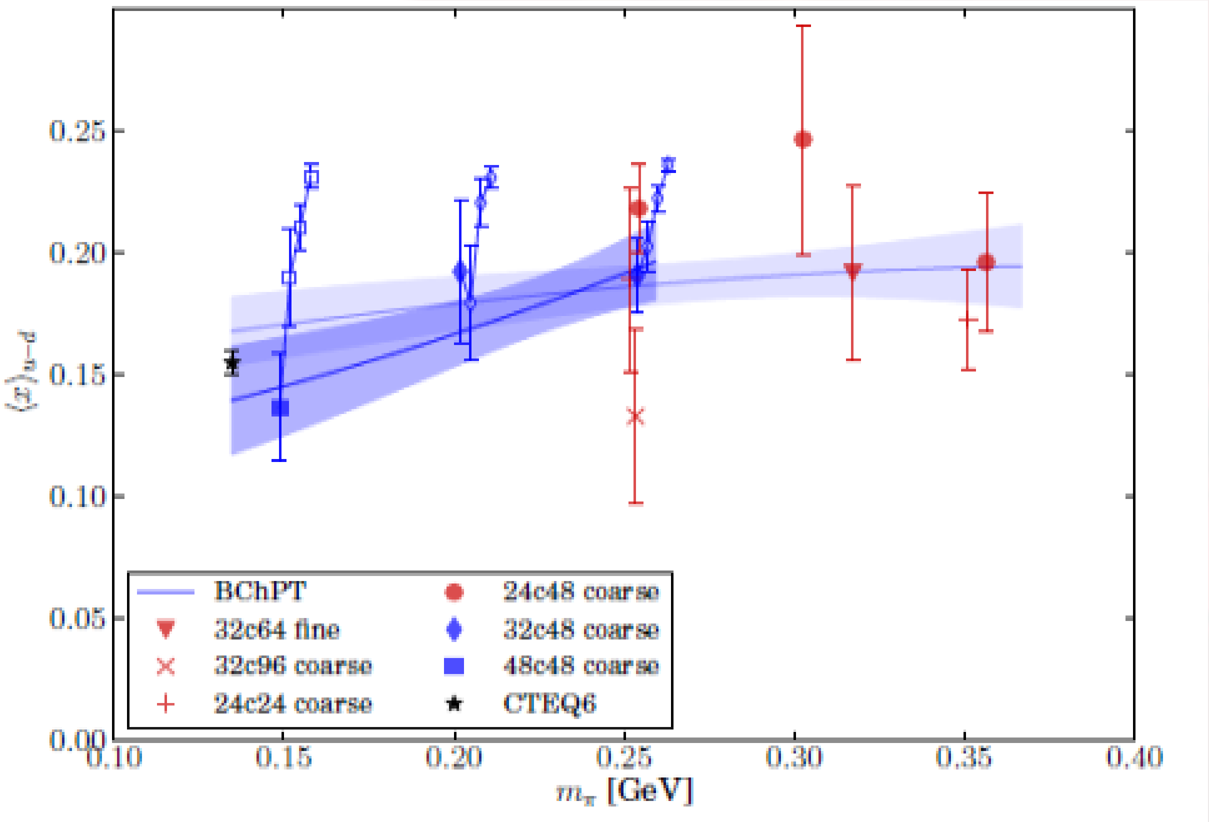
- The only known rigorous framework for *ab-initio* calculation of the structure of protons and neutrons with controllable errors.
- After decades of effort, one can finally calculate nucleon properties with dynamical fermions at physical pion mass!



Berkeley Lab

Nucleon Structure from Lattice QCD

J.R. Green et al, 2012 & 2014

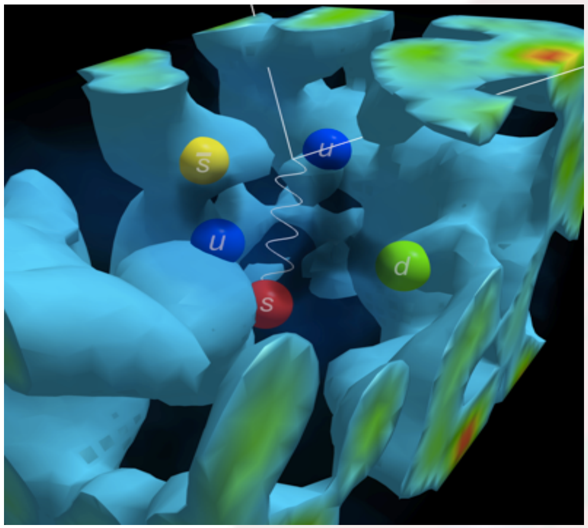


Nearly physical pion mass m_π=149MeV

Quark momentum fraction

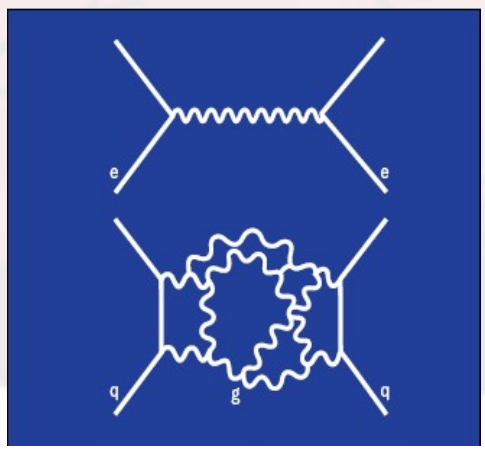
$$\langle x \rangle_{u-d} = \int dx\, x\, (u + \bar{u} - d - \bar{d})$$

Fundamental Understanding of the Nucleon Structure in QCD



Lattice QCD

EXP. Measurements



Theory/ Phenomenology