A mechanism of neoclassical tearing modes onset by drift wave turbulence

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Outline

- Motivation
- A mechanism of neoclassical tearing modes onset by drift wave turbulence
- Summary

Motivation

- **Some critical problems in ITER:**

【Nucl. Fusion, *Progress in the ITER Physics basis* (2007)】

  - **MHD instabilities: neoclassical tearing modes (degrade confinement capability of plasma, even lead to disruption), and so on**
  - **Plasma confinement and transport (anomalous transport induced by turbulence)**
  - **……**

- **NTMs and turbulence can interact strongly.**

Interaction between NTMs and Turbulence

- Turbulence modulated by TM or NTM: *Yu, NF 1992*; *Bardóczi,PRL 2016; Sun, PPCF 2018; ….*

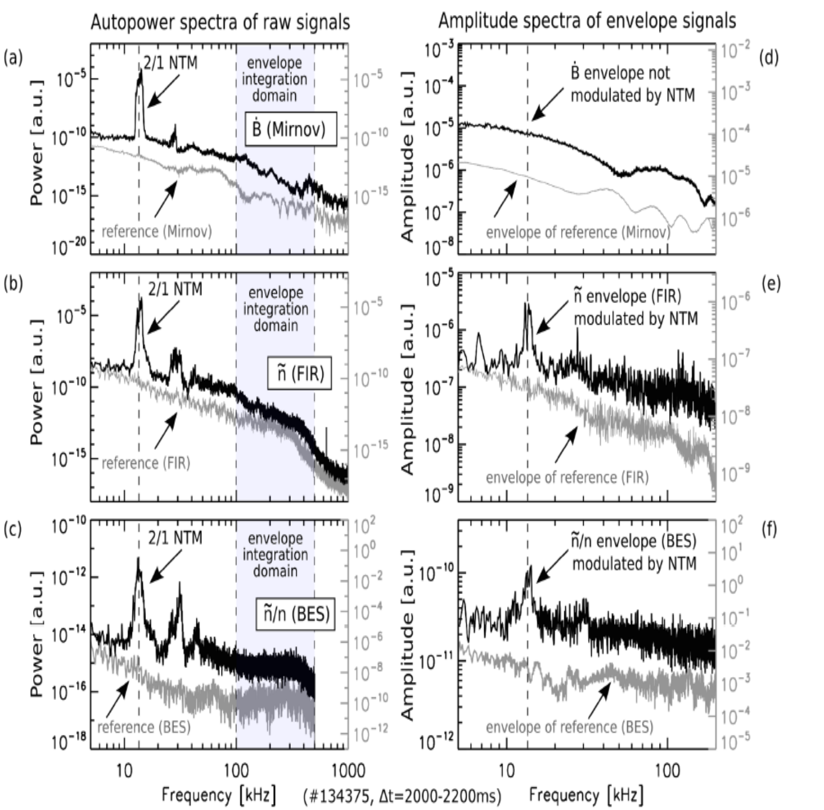


FIG. 2. Power spectrum of (a) $\tilde{B}$ (Mirnov), (b) $\tilde{n}|_{\mathrm{FIR}}$, and (c) $\tilde{n}/n|_{\mathrm{BES}}$. Amplitude spectrum of the corresponding envelope signals: (d) envelope of $\tilde{B}$, (e) envelope of $\tilde{n}|_{\mathrm{FIR}}$, and (f) envelope of $\tilde{n}/n|_{\mathrm{BES}}$. Note that the magnetic and FIR reference signals (light color) were measured before NTM onset, while the reference BES signal was measured during the NTM but further out (17 cm away from R_s showing the radial and temporal localization of the NTM.

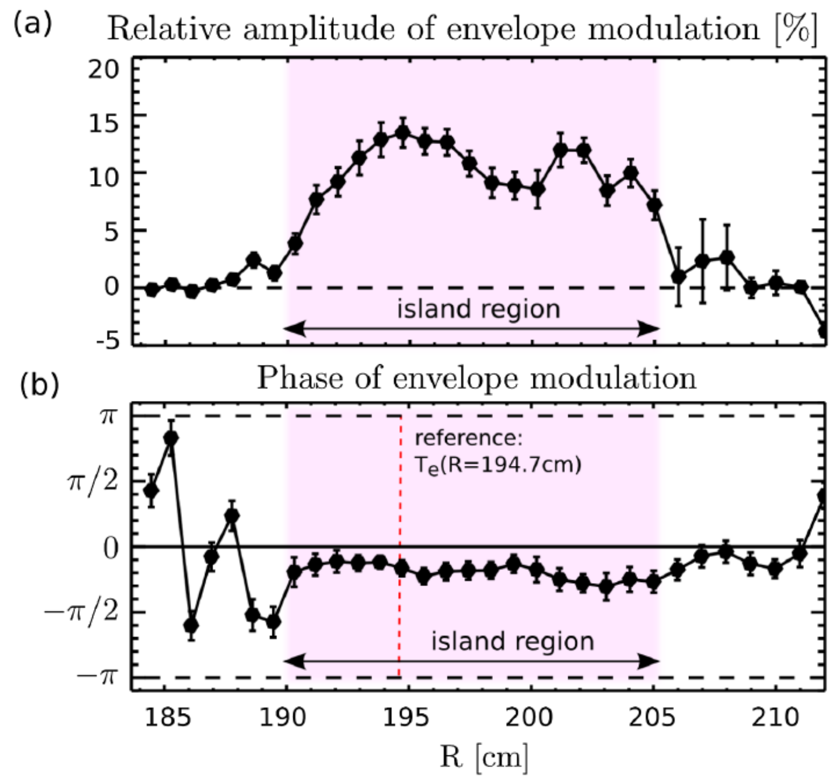
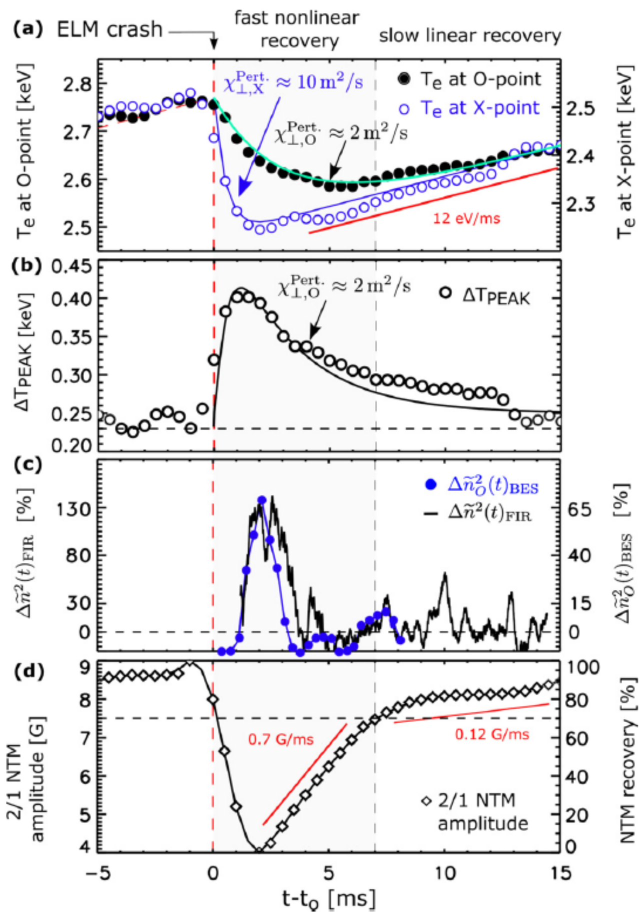


FIG. 4. (a) Modulation amplitude of broadband turbulence relative to the mean fluctuation levels (BES) and (b) phase of modulation relative to $T_e(R < R_s)$ (134375).

Bardóczi,PRL 2016

Interaction between NTMs and Turbulence

◆ TM or NTM driven by turbulence: *Kaw, PRL 1979; Itoh, PRL 2003; McDevitt, POP 2006; Sen, NF 2009; Wang, POP 2009; Muraglia, NF 2017; Ishizawa, POP 2013; Isayama, JPFR 2013; Bardóczi,POP 2017; ....*



➢ Turbulence accelerated the recovery of NTM island.

FIG. 5. (a) Fast T_e drop at X-point is followed by slower drop at O-point (due to different local $\chi_\perp$) leading to an increase of T_e peak at the O-point shown in (b). (c) low-k fluctuations across the island increase 2–4 ms after an ELM crash as seen from $\tilde{n}^2_{FIR}$ (#134375) as well as locally at the O-point as seen from $\tilde{n}^2_{BES}$ (#134360). (d) 2/1 NTM amplitude decreases and recovers in sync with the T_e peak. (Solid lines are analytical solutions of the transient heat transport model [Eq. (6)] on (a) and (b).

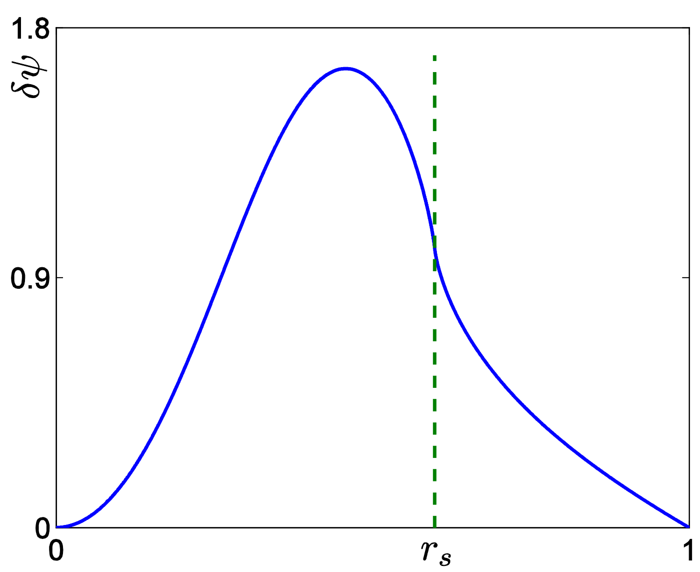
Bardóczi,POP 2017

The basic physics of tearing modes

- **Away from rational surface, $\delta\psi$ is determined by the ideal MHD equations. It has a discontinuous derivative at rational surface.**
- **At the singular layer, $\delta\psi$ is determined by resistive MHD Eqs.**
- **By matching the solutions of outer and inner region, the dispersion relation can be obtained.**

$$\Delta'\delta\psi = 2\mu_0\int_{0^-}^{0^+} dx \oint d\xi J_{//}\cos\xi,$$

$$\Delta' = \frac{1}{\delta\psi}\left[\left.\frac{d\delta\psi}{dr}\right|_{r_s^+} - \left.\frac{d\delta\psi}{dr}\right|_{r_s^-}\right].$$



Basic physics of neoclassical tearing modes

- The free energy source for the instability is the bootstrap current, pressure driven magnetic island

- Slowing evolving equilibrium governed by neoclassical Ohm's law:

$$<\mathbf{E}\cdot\mathbf{B}>=\eta<\mathbf{J}\cdot\mathbf{B}>-\frac{1}{ne}<\mathbf{B}\cdot\nabla\cdot\boldsymbol{\pi}_e>,$$

$$\rightarrow J_{//}=J_{ohm}+J_{bs}$$

- Modified Rutherford equation:

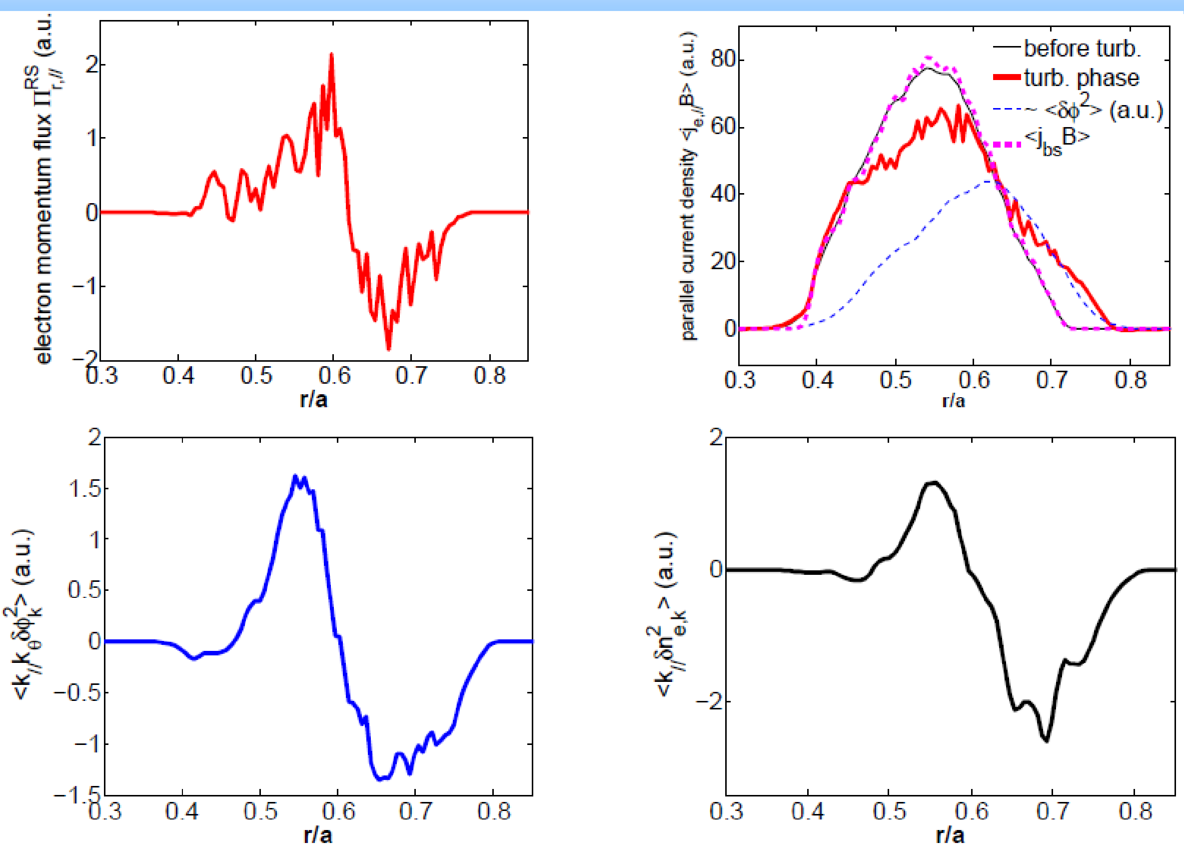
$$\frac{\tau_r}{r_s^2}\frac{dw}{dt}=\Delta'+\Delta'_b+\ldots,$$

$$\Delta'_b=\frac{D_{nc}}{w}=-a_1\sqrt{\varepsilon}\frac{dp}{dr}\frac{q}{q'B_\theta^2}=\frac{<\mathbf{J}\cdot\mathbf{B}>_{bootstrap}}{<B^2>}\frac{V'<B^2/|\nabla\psi|^2>}{dq/d\psi}$$

Turbulence driven current

- Mechanisms:
  - Residual electron stress: can drive electron momentum flux and lead to a turbulence-driven current (acting like residual ions stress) (*Garbet X. et al 2014; Wang W.X. et al 2012)*
  - Turbulence acceleration: relies on the exchange of momentum between ions and electrons, which can also accelerate electrons and drive a current. (*Garbet X. et al 2014;Wang W. X. Et al 2012*)
  - Resonant scattering: establish an equilibrium between trapped and passing electrons due to resonant scattering by turbulence, and drive a mean current. *(McDevitt, et al, 2017)*
  - ✓ The Previous two mechanisms are similar to those of ions, resulting from the symmetry breaking of turbulence spectrum.

Turbulence driven current



Wang,APS 2017

- Current profile significantly modified
- Finite $\langle k_{\parallel} \rangle$ is needed for both parallel acceleration and residual stress
- $k_{\parallel}$ symmetry breaking is caused by turbulence intensity

Heuristic interpretation

- With turbulence-driven current, the perturbed modified Ohm's law

$$\delta J_{//} = \sigma_{sp}\delta E_{//} + \delta J_{bs} + \delta J_{tur}$$

- δJ_{tur} is the perturbed turbulence-driven current, affected by the large scale ExB drift flow related to NTMs.

$$\Rightarrow \quad \delta J_{tur} \propto \delta\phi^{ntm}$$

where $\delta\phi^{ntm}$ is the electrostatic potential of NTMs.

- Hence, turbulence-driven current may affect NTMs via parallel Ohm's law.

- Turbulence also can affect NTMs by changing the transport coefficients, like viscosity, thermal diffusivity…

A mechanism of NTMs onset by drift wave turbulence

- Given magnetic field:

$$\mathbf{B} = I\nabla\zeta + \nabla\zeta \times \nabla(\psi + \delta\psi), \delta\psi = \delta\hat{\psi}\cos\xi, \xi = m\theta - n\zeta$$

- The matching between outer region and inner region:

$$\Delta' = \frac{1}{\delta\hat{\psi}} \int_{0^-}^{0^+} \frac{\partial^2 \delta\hat{\psi}}{\partial x^2} dx = \frac{2}{\delta\hat{\psi}} \frac{4\pi R}{c} \oint \frac{d\xi}{2\pi} \int_{0^-}^{0^+} dx \delta J_{//} \cos\xi$$

introducing

$$\Omega = -\delta\psi / \delta\hat{\psi} + \frac{2}{w^2} x^2 = -\delta\psi / \delta\hat{\psi} + \frac{2}{w_\chi^2} (\psi - \psi_s)^2, w^2 = 4\left(\frac{\partial\psi}{\partial r}\right)^{-1} \frac{q_s \delta\hat{\psi}}{q_s'}, w_\chi = w\left(\frac{\partial\psi}{\partial r}\right)$$

One can obtain

$$\Delta' = \frac{16 q_s}{w^2 \psi' q_s'} \frac{4\pi R}{c} \frac{w}{2\sqrt{2}} \int_{-1}^{\infty} d\Omega < \delta J_{//} \cos\xi >$$

where

$$< \ldots > = \oint \frac{d\xi}{2\pi} \frac{(\ldots)}{\sqrt{\Omega + \cos\xi}}$$

denotes the flux surface average.

The modified Ohm's law

◆ Drift kinetic equation for mean distribution:

$$v_{//}\mathbf{b}\cdot\nabla\overline{F_e}+\mathbf{v}_d\cdot\nabla\overline{F_e}-ev_{//}E_{//}\frac{\partial\overline{F_e}}{\partial\varepsilon}+\overline{\delta\mathbf{v}_e^{tur}\cdot\nabla\delta F_e^{tur}}-\overline{ev_{//}\delta E_{//}^{tur}\frac{\partial\delta F_e^{tur}}{\partial\varepsilon}}=C(\overline{F_e})$$

where $f_e=\overline{F_e}+\delta F_e^{tur},\overline{\delta F_e^{tur}}=0,$ and only electrostatic fluctuation is considered.

separating $\overline{F_e}=F_b+F_c, F_b$ is caused by magnetic drift,

$$\oint\frac{qRd\theta}{v_{//}}\left[\overline{\delta\mathbf{v}_E^{tur}\cdot\nabla\delta F_e^{tur}}-\overline{ev_{//}\delta E_{//}\frac{\partial\delta F_e^{tur}}{\partial\varepsilon}}-ev_{//}E_{//}\frac{\partial F_M}{\partial\varepsilon}\right]=\oint\frac{qRd\theta}{v_{//}}C(F_c)$$

Further, the Krook model is assumed, as $C(F_c)=-\nu_e F_c$,

$$F_c=-\frac{1}{\nu_e}\left[\overline{\delta\mathbf{v}_E^{tur}\cdot\nabla\delta F_e^{tur}}-\overline{ev_{//}\delta E_{//}^{tur}\frac{\partial\delta F_e^{tur}}{\partial\varepsilon}}-ev_{//}E_{//}\frac{\partial F_M}{\partial\varepsilon}\right]$$

Here, electrons are assumed to be passing, the scattering effect and collisionless bootstrap current due to turbulence are not considered.

The modified Ohm's law

- The Ohm's law including electrostatic turbulence:

$$J_{//} = \sigma_{sp} E_{//} + J_{bs} + J_{tur}, \quad J_{tur} = \frac{e}{m_e \nu_e}\left(\frac{1}{r}\frac{\partial}{\partial r} r\Pi_{//e} + M_{//e}\right)$$

$\Pi_{//e} = 2\pi m_e \int d\mu B_0 / m_e \int dv_{//} v_{//} \overline{\left(\delta \mathbf{v}_E^{tur} \cdot \nabla r \delta F_e^{tur}\right)}$ is an electron momentum,

$M_{//e} = 2\pi e \int d\mu B_0 / m_e \int dv_{//} \overline{\delta E_{//}^{tur} \delta F_e^{tur}}$ from electron-ion momentum exchange.

$$\Rightarrow \Pi_{//e} = -\chi_\phi \frac{\partial \bar{u}_{//,e}}{\partial r} + V\bar{u}_{//,e} + \pi_{//e}$$

The first term is anomalous electron viscosity, the second term is a pinch of electron momentum, and the last term refers to electron residual stress.

Next, we focus on the last term.

The modified Ohm's law

- The expression of perturbed distribution can be obtained from linearized drift equation:

$$\delta F_{e,m,n}^{tur} = -\frac{\omega - \omega_e}{\omega - k_{//} v_{//}} F_{Me} \frac{e\delta\phi_{m,n}^{tur}}{T_e}$$

where the effects of magnetic drift and collision are not considered.

Keeping transit resonance, one can obtain

$$\pi_{//e} = \sqrt{\pi/2} n_e m_e \sqrt{m_e/m_i} \rho_i \sum_{m,n} \frac{m}{r} \frac{\omega}{k_{//}} \frac{\omega - \omega_{*e}}{|k_{//}|} \exp\left(-\frac{m_e}{2T_e}\frac{\omega^2}{k_{//}^2}\right) \left|\frac{e\delta\phi_{m,n}^{tur}}{T_e}\right|^2$$

$$M_{//e} = \sqrt{\pi/2} n_e m_e c_{se} \sum_{m,n} \exp\left(-\frac{m_e}{2T_e}\frac{\omega^2}{k_{//}^2}\right) \frac{k_{//}}{|k_{//}|} (\omega - \omega_{e*}) \left|\frac{e\delta\phi_{m,n}^{tur}}{T_e}\right|^2$$

The evolution of turbulence in the presence of NTMs

- A wave kinetic equation (WKE) for the drift wave action density:

$$\frac{\partial N_k}{\partial t}+\frac{\partial}{\partial \mathbf{k}}\left(\omega_k+\mathbf{k}\cdot\mathbf{v}^{ntm}\right)\cdot\frac{\partial N_k}{\partial \mathbf{x}}-\frac{\partial}{\partial \mathbf{x}}\left(\mathbf{k}\cdot\mathbf{v}^{ntm}\right)\cdot\frac{\partial N_k}{\partial \mathbf{k}}=S,\ N_k=\left(1+k_\perp^2\rho_i^2\right)|\,e\delta\phi_{m,n}^{tu}/T_e\,|^2$$

ω_k is frequency of drift wave,

$\mathbf{v}^{ntm}=c\mathbf{b}\times\nabla\delta\phi^{ntm}/B_0$ is the electrostatic flow of NTM.

$S=\gamma_k N_k-\Delta\omega_k N_k^2$ is the source term, where the first term denotes the linear drive of drift waves in the presence of NTMs, the second term represents the nonlinear like-scale interaction.

Here, it is assumed that the self-interaction of small-scale turbulence fields is small compared to the interaction between turbulence and NTMs.

The evolution of turbulence in the presence of NTMs

Considering small deviation from the equilibrium drift wave spectrum,

$$\frac{\partial \delta N_k}{\partial t}+\frac{\partial \omega_k}{\partial \mathbf{k}}\cdot\frac{\partial \delta N_k}{\partial \mathbf{x}}+\frac{\partial}{\partial \mathbf{k}}\left(\mathbf{k}\cdot\mathbf{v}^{ntm}\right)\cdot\frac{\partial N_{k0}}{\partial \mathbf{x}}-\frac{\partial}{\partial \mathbf{x}}\left(\mathbf{k}\cdot\mathbf{v}^{ntm}\right)\cdot\frac{\partial N_{k0}}{\partial \mathbf{k}}=\gamma_k \delta N_k$$

$$\Rightarrow \quad \delta N_k \sim \frac{\gamma_k}{\gamma_k^2+(\mathbf{l}\cdot\mathbf{v}_g)^2} k_\theta \frac{\partial N_{k0}}{\partial k_r}\frac{c}{B_0}\frac{\partial^2 \delta\phi^{ntm}}{\partial r^2},\ \mathbf{v}_g=\frac{\partial \omega_k}{\partial \mathbf{k}}$$

Then, the turbulence-driven current perturbed by NTMs:

$$\delta J_{tur} \sim \sqrt{\pi/2}\,\frac{n_e e^2}{m_e \nu_e}\mu_{//e} a^2 \frac{\partial^3 \delta\phi^{ntm}}{\partial r^3},$$

$$\mu_{//e}=\left(\frac{m_e}{m_i}\right)^{3/2}\int d\mathbf{k}\,\frac{k_\theta^2\rho_i^2}{k_{//}\,|k_{//}|}\frac{\rho_i}{L_n}\frac{\omega-\omega_{*e}}{\omega_{*e}}\exp\left(-\frac{m_e}{2T_e}\frac{\omega^2}{k_{//}^2}\right)\frac{1}{(1+k_\perp^2\rho_i^2)^2}k_\theta\frac{\partial N_{k0}}{\partial k_r}\frac{\gamma_k \omega}{\gamma_k^2+(\mathbf{l}\cdot\mathbf{v}_g)^2}$$

The current from $M_{//e}$ is not included, since it is an odd function, and has no effect on NTMs.

The evolution of NTMs

The evolution of magnetic island

$$\frac{8\pi}{c^2}\frac{I_1}{\eta}\frac{dw}{dt}=\Delta'+\frac{G_1\sqrt{\varepsilon_s}r_s}{sL_n}\frac{\beta_\theta}{w}\left(\frac{w^2}{w^2+w_\chi^2}-\frac{w_{pol}^2}{w^2}+\frac{\sigma^2 w_{tur}^2}{w^2}\right)$$

$$w_{pol}^2=\sqrt{G_2}\left(\frac{r_s}{sL_n}\right)^{1/2}\sqrt{\varepsilon_s}\rho_{\theta i},\quad w_{tur}^2=\frac{G_3}{\sqrt{\varepsilon_s}}\frac{\tau_R}{\tau_A}\frac{r_s d_i}{q_s}|\mu_{//,e}|$$

σ denotes the sign of the turbulence intensity gradient or the shear flow gradient.

Considering the drift wave turbulence,

$$w_{tur}^2\sim\frac{G_3}{\sqrt{\varepsilon_s}}\frac{\tau_R}{\tau_A}\left(\frac{m_e}{m_i}\right)^{3/2}\frac{r_s^3 d_i\rho_{\theta i}^2}{q_s s^2 L_n|L_I|a^2}I_{tur}$$

where $k_\theta\rho_i\sim 1$ and the turbulence mode width $w_{tu,k}\sim\rho_i$ are chosen.

$$I_{tur}=\sum_{m,n}|e\delta\phi_{m,n}^{tur}/T_e|^2,L_I=\left(d\ln I_{tur}/dr\right)^{-1}$$

The effect of turbulence is similar to that of neoclassical polarization current. It would change the onset threshold of NTMs.

The onset of NTMs

The onset threshold of NTMs

$$\beta_\theta^{onset} = -r_s\Delta'\left(G_1\sqrt{\varepsilon_s}\frac{r_s}{sL_n}\right)^{-1}\frac{w_{pol}}{r_s}\left[\frac{\hat{w}_{seed}}{\hat{w}_{seed}^2 + w_\chi^2/w_{pol}^2} - \frac{1}{G_1\hat{w}_{seed}^3}\left(1-\frac{\sigma w_{tur}^2}{w_{pol}^2}\right)\right]^{-1}, \quad \hat{w}_{seed} = w/w_{pol}$$

- The effect of turbulence-driven current on onset threshold of NTMs depends on the ratio w_{tur}^2/w_{pol}^2.
  - The effect depends on the direction of turbulence intensity gradient.
  - For typical values of tokamak, $w_{tur}^2/w_{pol}^2 \sim O(1)$,

    namely the effect of turbulence-driven current is significant.

If $\sigma > 0$, it enhances the onset threshold of NTMs.

If $\sigma < 0$, it reduces or overcomes stabilizing effect of neoclassical polarization current, and can trigger NTMs.

For the typical tokamak, like DIII-D, $R_0 = 1.7m, a = 0.61m, B = 1.6T$,

Given $T_i = T_e = 2keV, n_i = 2\times10^{19} m^{-3}, r_s\Delta' = -3, q_s = 2, s = 1, \varepsilon_s^{1/2} = 0.5, L_n = |L_I| = 0.5a$,

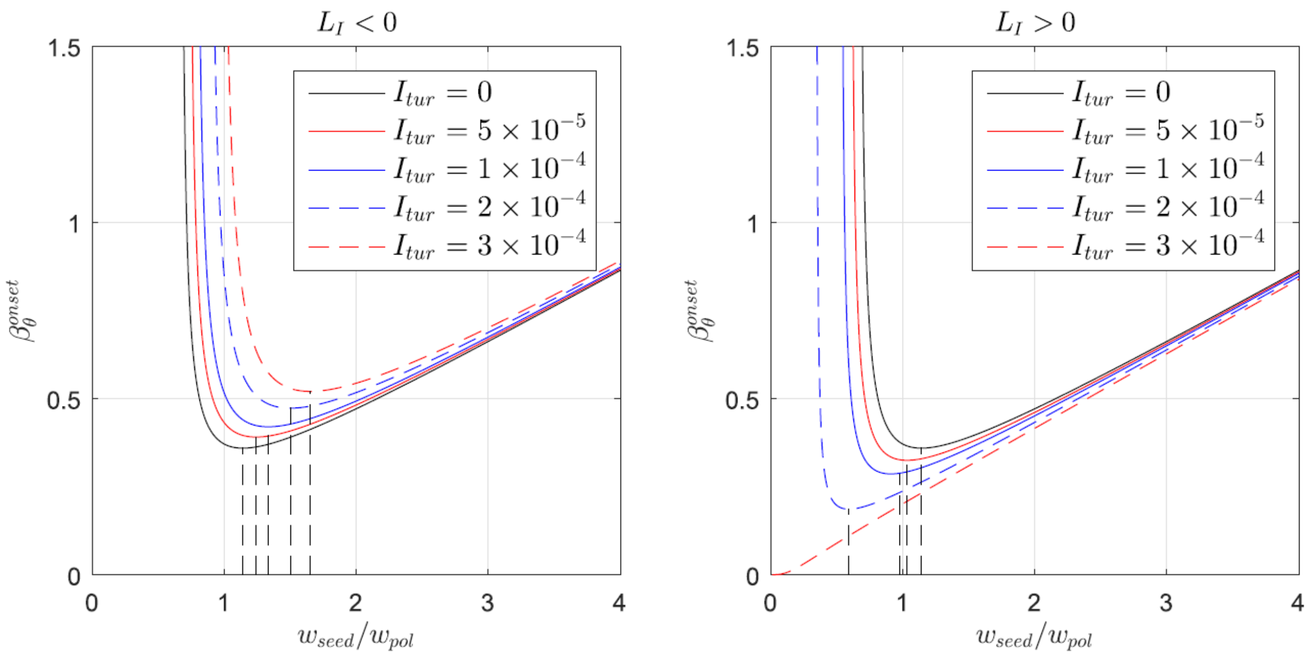


Figure 1. The dependence of $\beta_\theta^{\mathrm{onset}}$ on $w_{\mathrm{seed}}/w_{\mathrm{pol}}$ for $L_I < 0$ and $L_I > 0$, respectively.

➢For $L_I < 0, \beta_\theta^{onest}, w_c$ increases with I_{tur}, namely the turbulence-driven current plays a stabilizing role, and enhances the onset threshold.

The onset of NTMs_*Nucl. Fusion 59, 026009(2019)*

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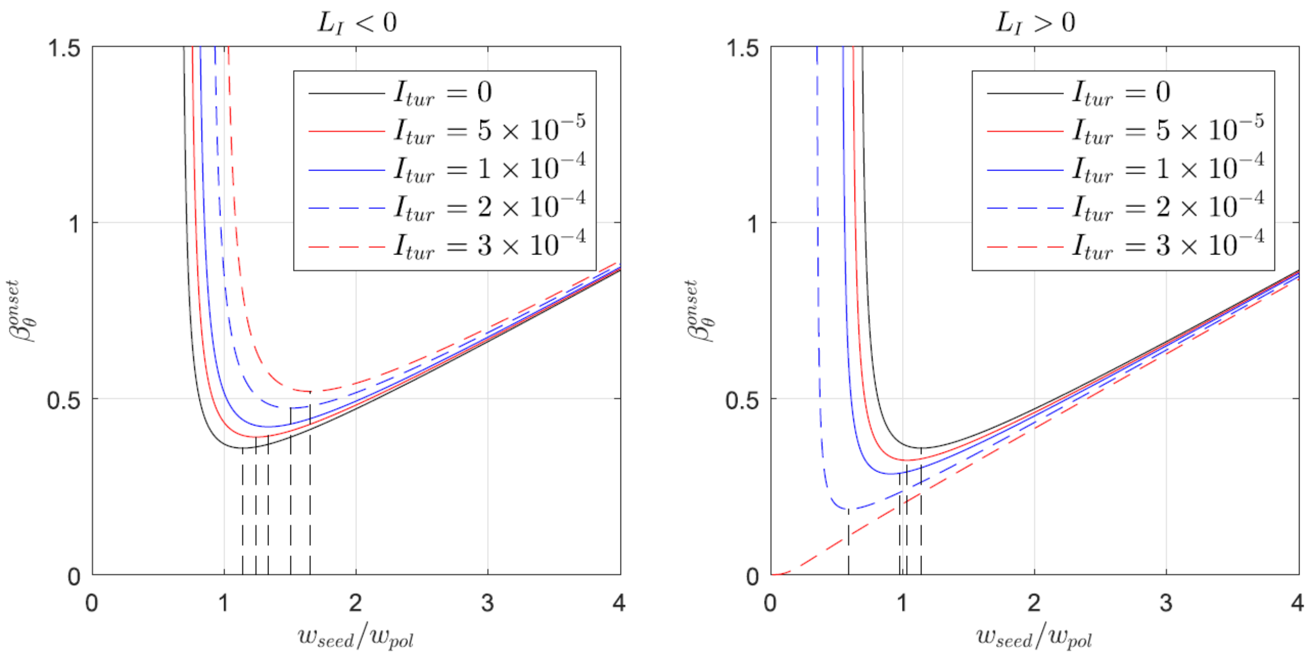

Figure 1. The dependence of $\beta_\theta^{\mathrm{onset}}$ on $w_{\mathrm{seed}}/w_{\mathrm{pol}}$ for $L_I < 0$ and $L_I > 0$, respectively.

- For $L_I > 0, \beta_\theta^{onest}, w_c$ decreases with I_{tur}, namely the effect is destabilizing, and cancels the stabilizing effect of neoclassical polarization current. It leads to a reduction of onset threshold and can trigger NTMs.

Summary and Discussion

- A new mechanism of turbulence-driven current on the onset threshold of NTMs is provided. The turbulence-driven current modifies Ohm's law, and alters the parallel current in the island.
- The effect of turbulence-driven current on NTMs is comparable to that of neoclassical polarization current.
- The onset threshold can be affected significantly by turbulence.
- The effect depends on the direction of turbulence intensity gradient at the resonance surface and the amplitude of turbulence.
- The triggering of NTMs by turbulence depends on type of turbulence and the symmetry breaking mechanism of turbulence spectrum.
- When the turbulence intensity gradient is positive, it may explain the recent experimental results in DIII-D. It also implies NTMs can appear without noticeable MHD events.

Summary and Discussion

- Here, we focus on the effect of turbulence-driven current on NTMs. The feedback effect of NTMs on turbulence is not studied, such as the modification of equilibrium profile by NTMs.
- It is valid for small island.
- It needs experiments to justify, while the experiments are hard to diagnose.
- It also needs a detailed simulation.

Thank you for your attention!