Whole-plasma simulation of self-consistent turbulent transport in a laboratory magnetosphere

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Outline

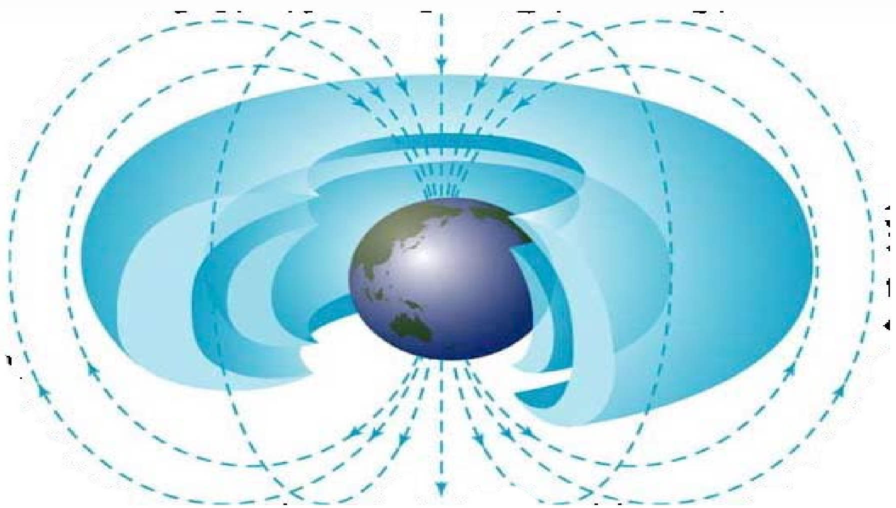
- Background
- Simulations of interchange mode and entropy mode of a dipole-confined plasma based on drift reduced model

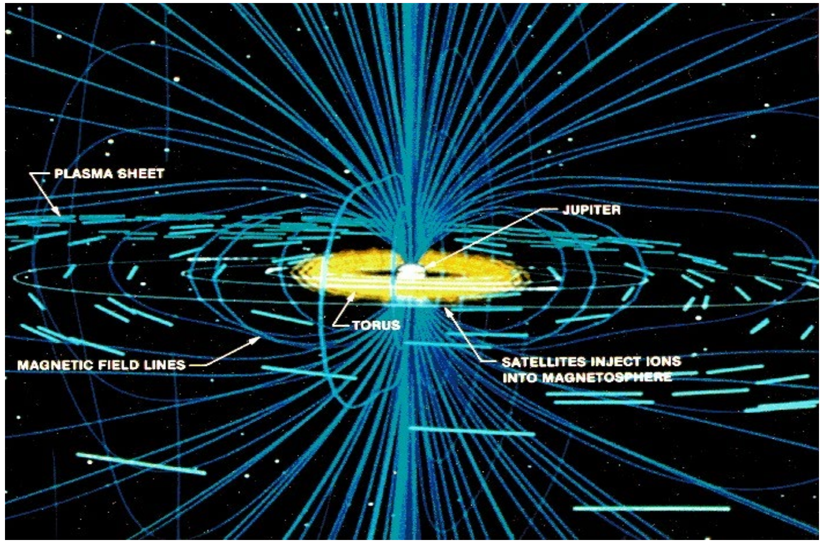
  Weike Ou, *et al*, PRE **101**, 021201(R) (2020)
- Conclusion

Planetary magnetosphere

- Unique confinement in a dipole magnetic field
- Centrally peaked profile of density and pressure
  - Gold(1959) $p \sim 1/\delta V^{\gamma}$
  - Melrose(1967) $\langle n \rangle \sim 1/\delta V$
- Possibility of achieving controlled fusion

A. Hasegawa, L. Chen, and M. Mauel, Nuclear Fusion (1990)

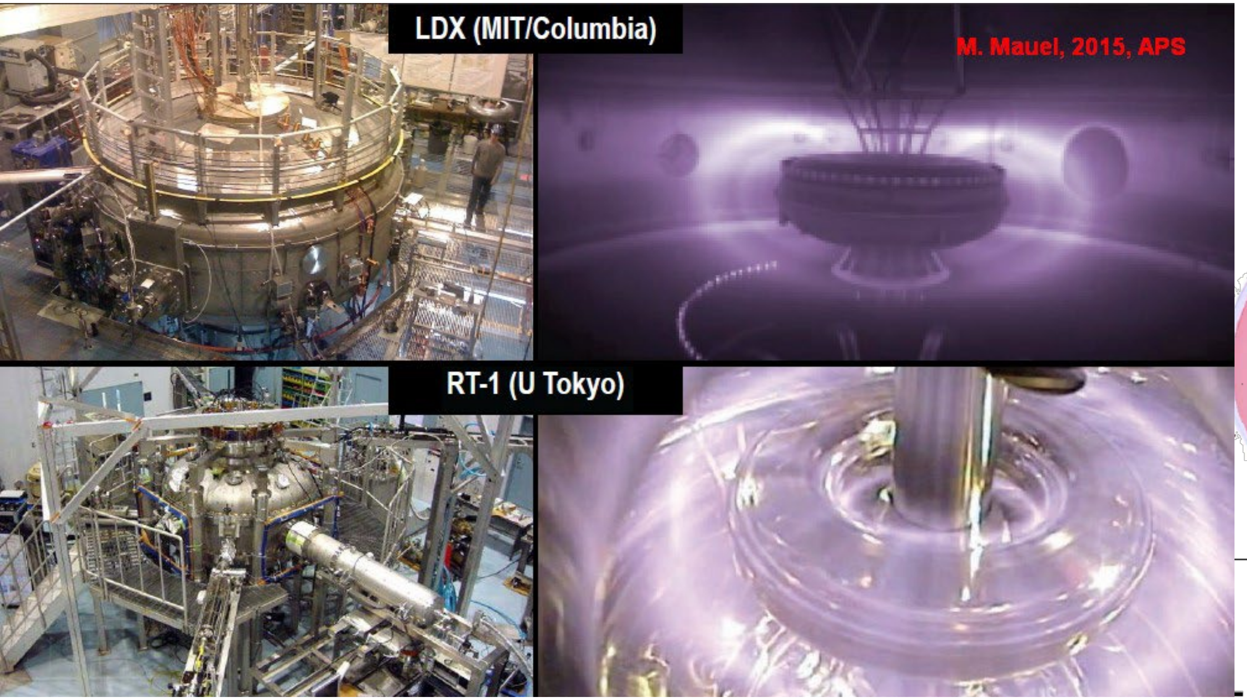


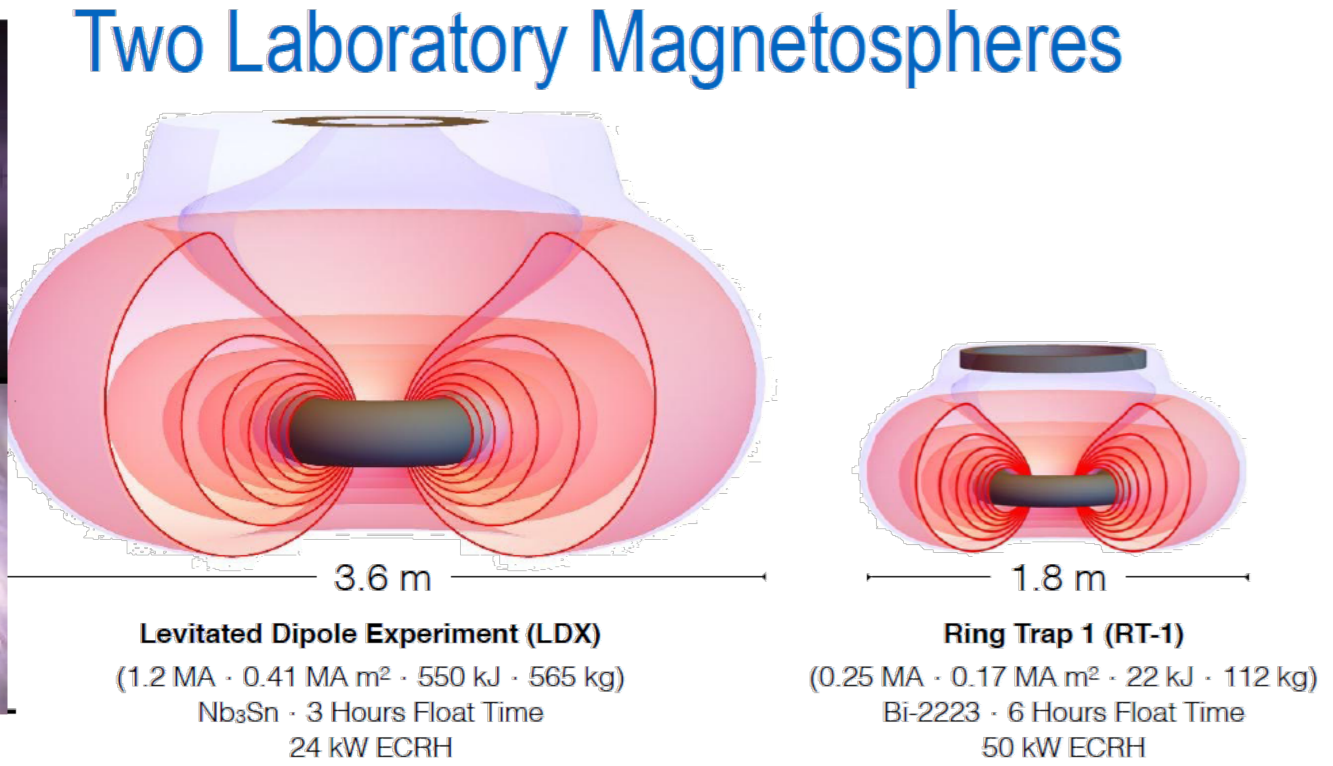


Earth(up) Jupiter (down) magnetosphere

Dipole-confined plasma device

CTX	LDX	RT-1	DREX
1990	2004	2006	2021
Columbia University	MIT	University of Tokyo	Harbin Institute of Technology

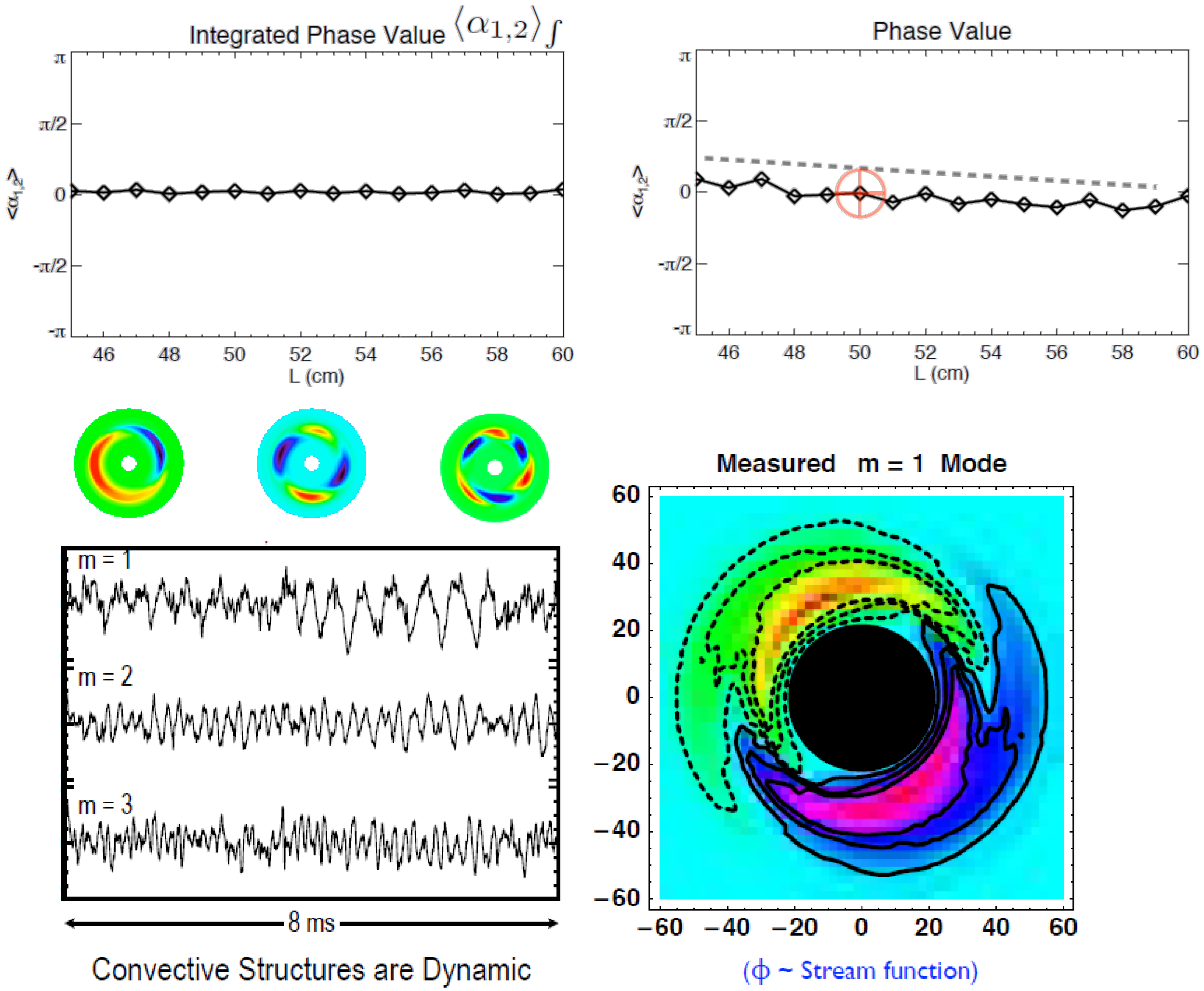




Mauel, APS, 2015

Observation in laboratory dipole plasma

- Dominant instability
  - electrostatic
  - $k_{\parallel} = 0$
- Global convective structure



Mauel, APS, 2014

Drift reduced model

$$\frac{\partial n}{\partial t} + \nabla \cdot n\boldsymbol{v}_e = 0$$

$$\nabla \cdot \boldsymbol{J} = \nabla \cdot en(\boldsymbol{v} - \boldsymbol{v}_e) = 0$$

$$\boldsymbol{E} + \boldsymbol{v}_e \times \boldsymbol{B} = -\frac{1}{en}\nabla p_e$$

$$\frac{\partial p_e}{\partial t} + \nabla \cdot (p_e \boldsymbol{v}_E) = -(\gamma - 1)p_e \nabla \cdot \boldsymbol{v}_E + \gamma \frac{2}{eB}\boldsymbol{b} \times \kappa \cdot \nabla(p_e T_e)$$

$$\boldsymbol{J} = \frac{1}{B^2}\boldsymbol{B} \times \nabla p_e + \frac{nm_i}{B^2}\boldsymbol{B} \times \left(\frac{d\boldsymbol{v}_E}{dt}\right)$$

Dipole geometry

- Dipole magnetic field

$$\boldsymbol{B} = \nabla\varphi \times \nabla\psi = \nabla\chi$$

- Normalization
  - Magnetic flux $\psi = x\psi_0$
  - Azimuthal angle $\varphi = y$
  - Time scale $\omega_0 = \rho_\star \frac{c_s}{L_0}$ □ $\rho_\star = \rho_i / L_0$
  - Spatial scale L_0

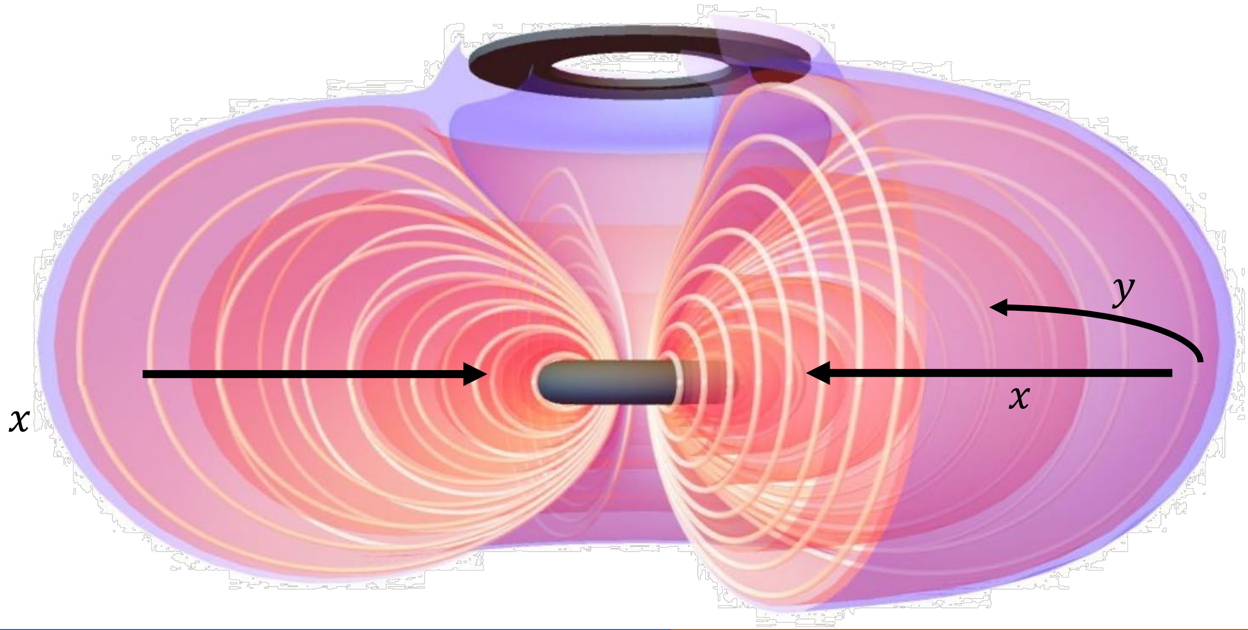
- Boundary conditions
  - Fixed boundary condition in x.
  - Periodic Boundary Conditions in y.

- Flux tube average

$$\langle A \rangle \equiv \frac{1}{\delta V(\psi)} \int \frac{A\,\mathrm{d}\chi}{B^2}$$

- Flux tube volume

$$\delta V(\psi) = \int \frac{d\chi}{B^2} \approx 0.91 \frac{M^3}{\psi^4}$$



Normalized Simulation Model

$$\frac{\partial N}{\partial t} - \frac{\partial}{\partial y}\left(N\frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial x}\left(N\frac{\partial \Phi}{\partial y}\right) = -4x^{4\gamma-5}\frac{\partial G}{\partial y}$$

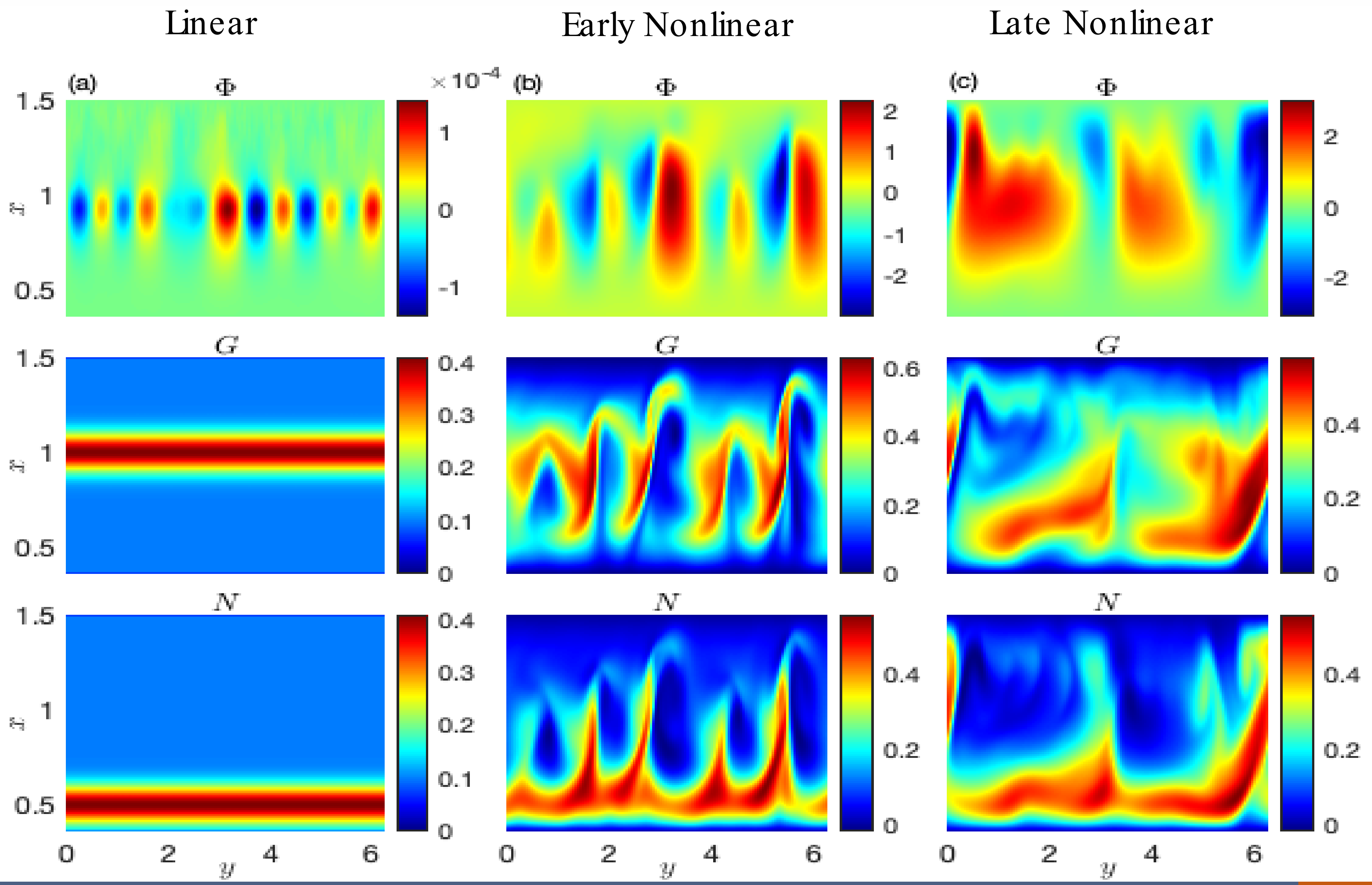
$$\frac{\partial G}{\partial t} - \frac{\partial}{\partial y}\left(G\frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial x}\left(G\frac{\partial \Phi}{\partial y}\right) = -4\gamma x^{4\gamma-5}\frac{\partial}{\partial y}\left(\frac{G^2}{N}\right)$$

$$\left(\frac{\partial w}{\partial t}\right) - \frac{\partial}{\partial y}\left(w\frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial x}\left(w\frac{\partial \Phi}{\partial y}\right) = -4x^{4\gamma-5}\frac{\partial G}{\partial y}$$

$$w = \rho_\star^2\left[\frac{\partial}{\partial x}\left(0.7x^{-2}N\frac{\partial \Phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(0.6x^{-4}N\frac{\partial \Phi}{\partial y}\right)\right]$$

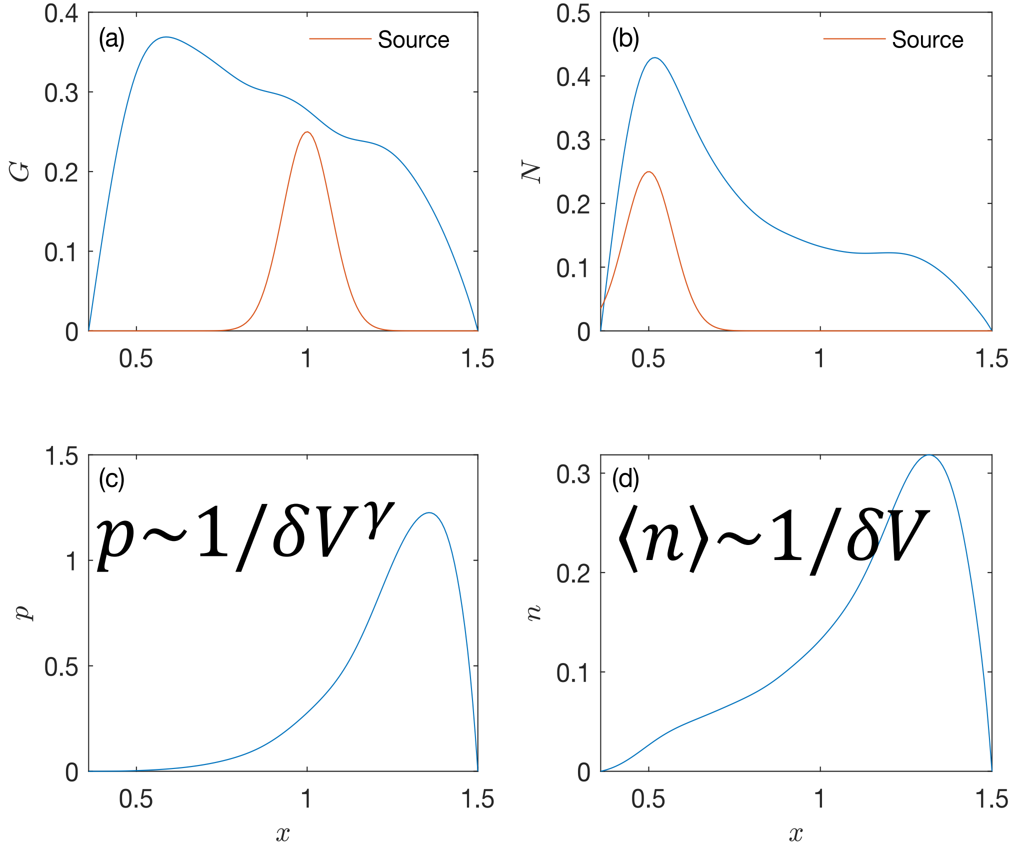
Define entropy function $G = p\delta V^{\gamma}$ and flux-tube particle number $N = n\delta V$

Nonlinear Simulation Results



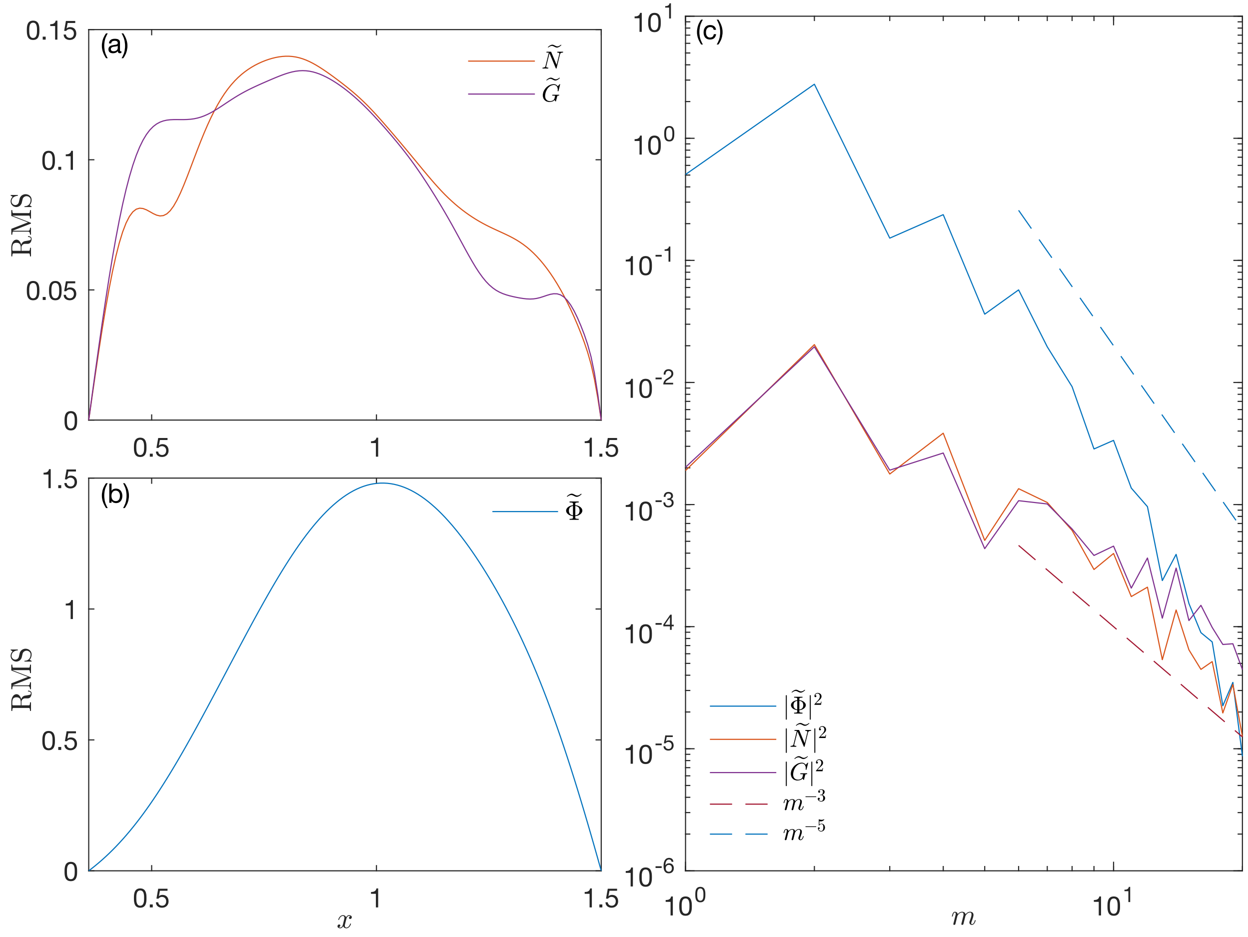
Structures of potential Φ, entropy G, particle number N.

Centrally peaked profile in a nonlinear state



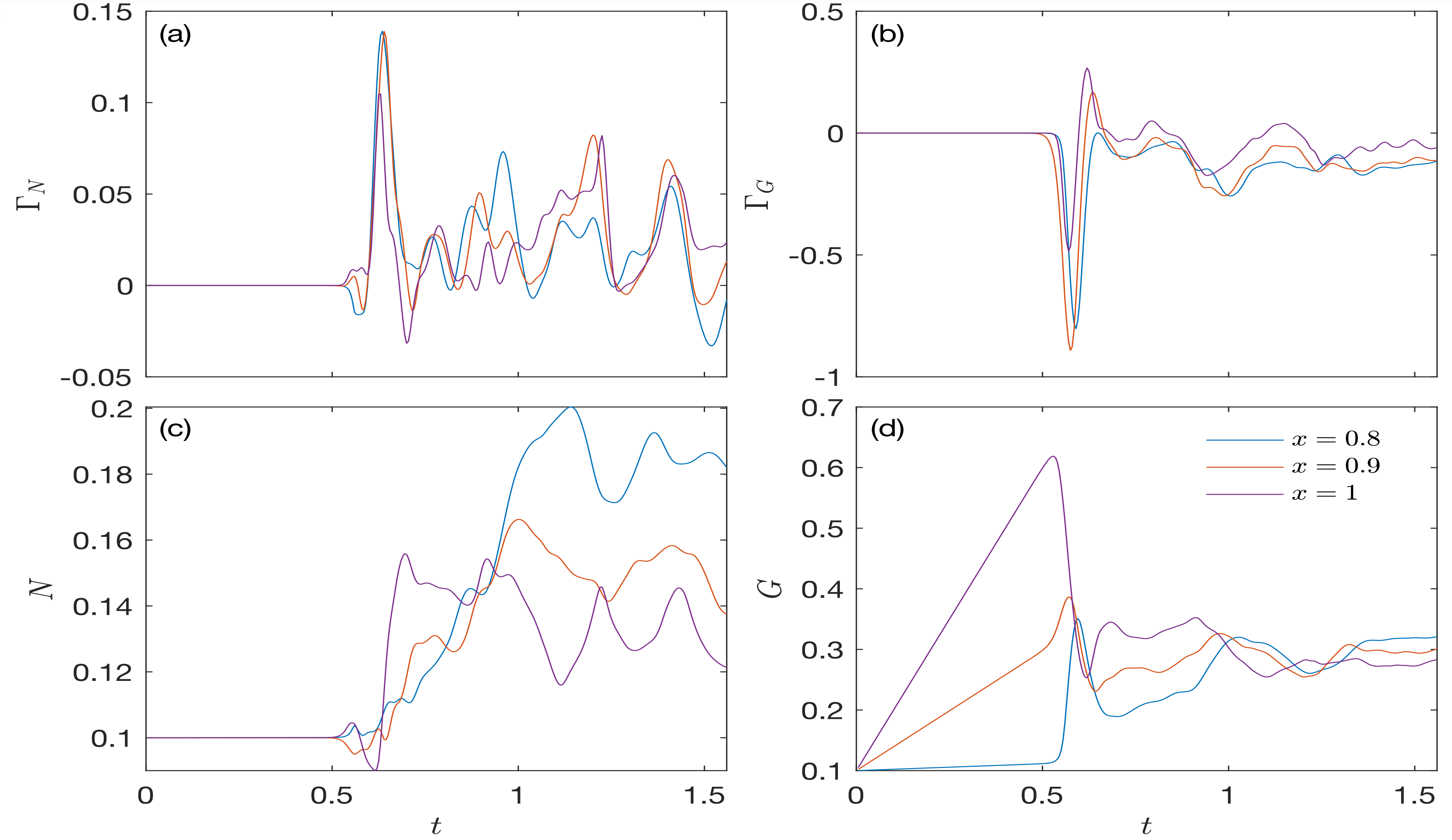
Time-averaged profiles of □ a□ entropy G□ b□ particle number N□ c□ pressure p□ d□ density $\langle n \rangle$.

$$\delta V \sim x^{-4}$$



Root-mean-square (rms) of□ a□ $\tilde{N}$□ $\tilde{G}$ and□ b□ $\tilde{\Phi}$ across radius.□ c□ Time-averaged fluctuation spectra of Φ□ N□ G in a nonlinear quasistationary state.

Inward transport of particles



Time evolution of azimuthally averaged□ a□ particle flux Γ_N□ b□ energy flux Γ_G□ c□ particle number N □ d□ entropy G at different locations.

Summary

- Global fluid simulations with flux-tube averaged model in dipole geometry.
- Turbulent convection of plasmas confined in a dipole field is characterized by global convective cells.
- Large-scale electrostatic potential fluctuations cause turbulent inward particle pinch in dipole plasmas.
- Centrally peaked density and pressure profiles are marginally stable to both interchange and entropy modes.