Analysis of energetic particle driven MHD modes on EAST

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10th US-PRC Magnetic Fusion Collaboration Workshop (online), 23-27 Mar., 2021

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- Motivation
- Kinetic-MHD model in NIMROD
- Functions & progress of AWEAC
- Fishbone/BAE/TAE on EAST
- Conclusions and future works

Motivation

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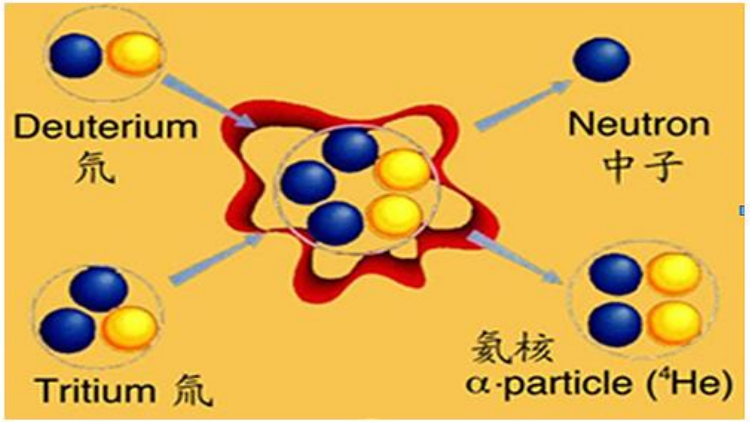
Fusion



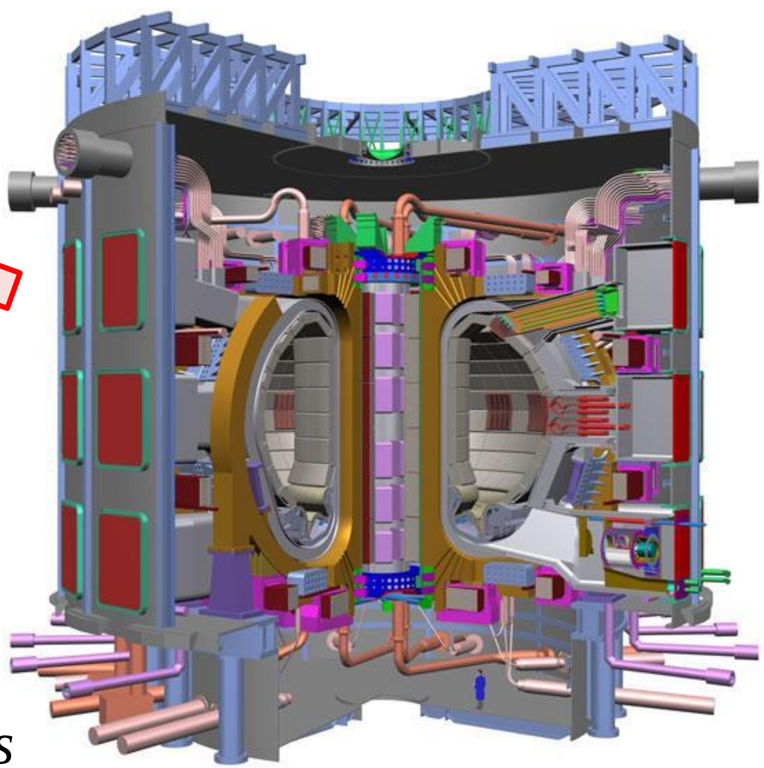
$0.9\times10^{6}m/s$ $\quad$ $12\times10^{6}m/s$

$$\mathrm{V}_{Ti} \ll \mathrm{V}_{A} < \mathrm{V}_{\alpha} \ll \mathrm{V}_{Te}$$

$8\times10^{6}m/s$ $\quad$ $59\times10^{6}m/s$

$$\mathrm{v}_{EP} \sim [3\times10^{6}m/s, 10\times10^{6}m/s]$$

ITER



Trapped EP Precession frequency、Core-ion diamagnetic frequency、Passing EP Radial drift velocity/circulating frequency ~~

Fishbone frequency/velocity;

EP toroidal velocity ~~ AEs phase velocity.

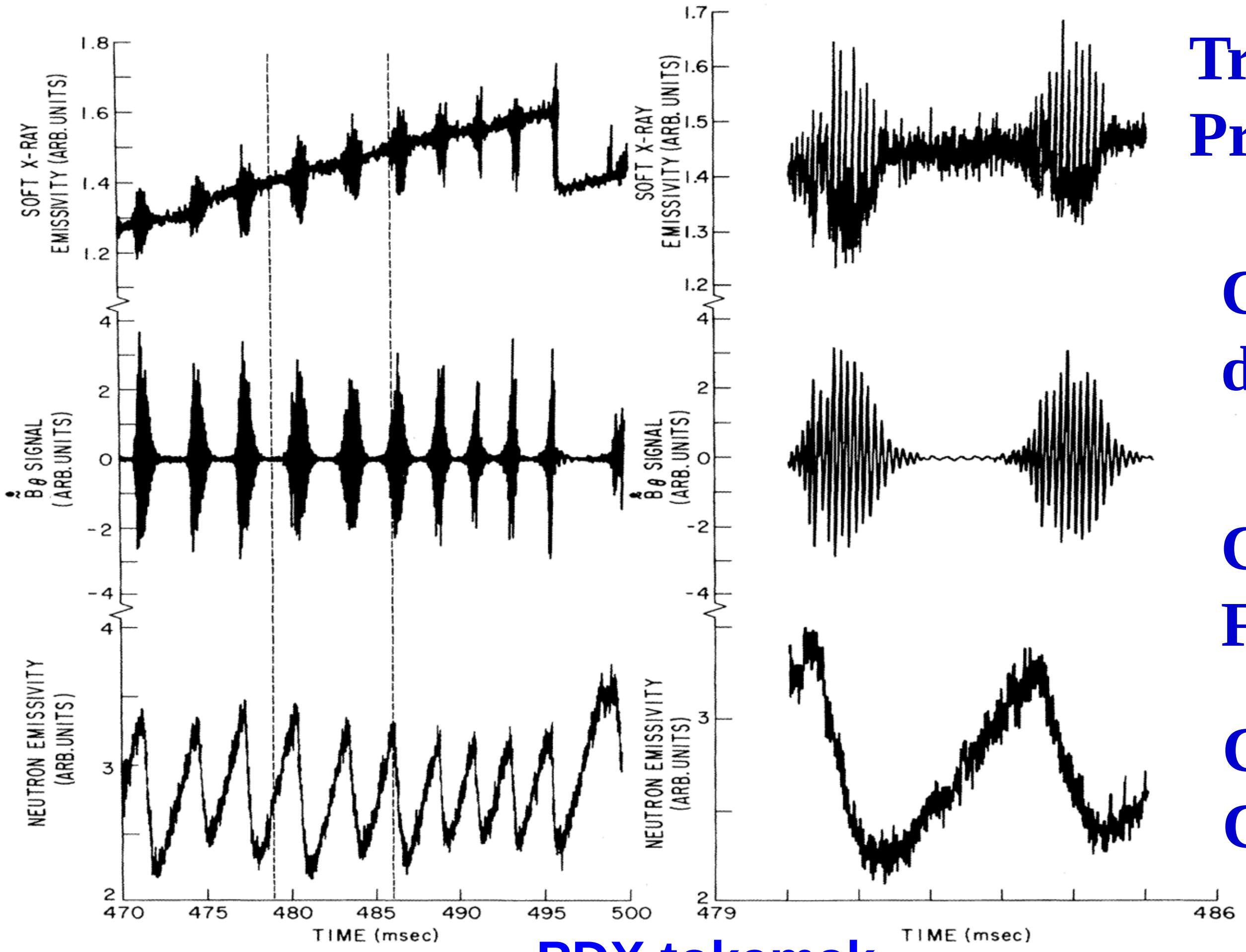
Heating & fusion -> energetic particle

Fishbone -> EP loss ~ 10%
Alfven Eigenmodes-> EP loss ~ 70%

1. Heating efficiency -> long pulse steady-state operation;
2. Reduction of EP destruction to FW -> long-lived component;
3. α particle heating -> self-sustained burning plasma.

Fishbone mode

McGuire et al PRL 1983



PDX tokamak

Trapped energetic ion
Precession frequency — Chen et al PRL 1984

Core-ion
diamagnetic motion — Coppi et al PRL 1986

Circulating energetic ion
Finite radial drift — Betti et al PRL 1993

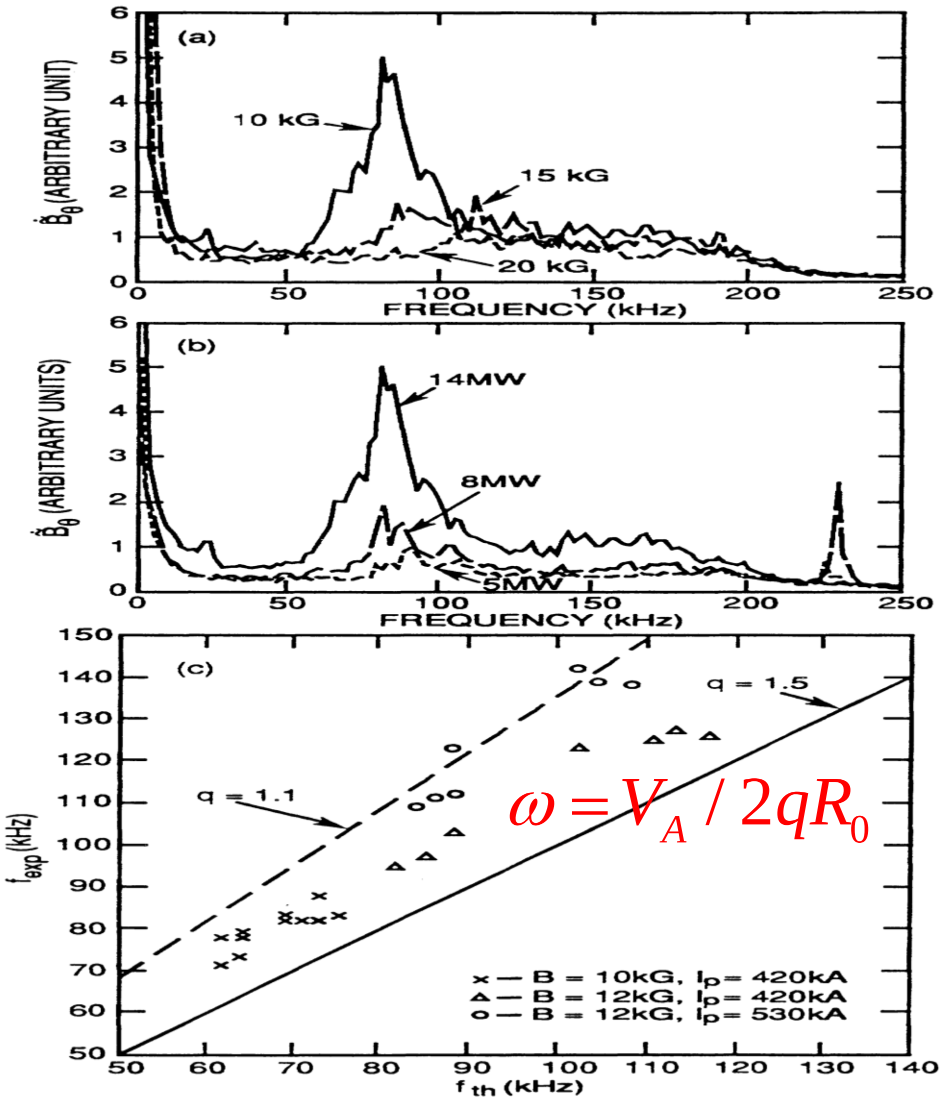
Circulating energetic ion
Circulating frequency — Wang et al PRL 2001

Superthermal electron — Wong et al PRL 2000, Macor et al PRL 2009

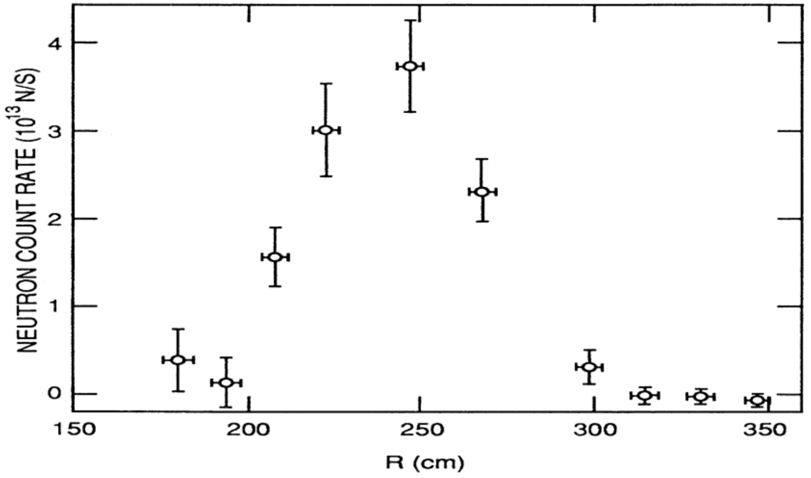
Energetic ion and electron can interact with kink mode to form fishbone.

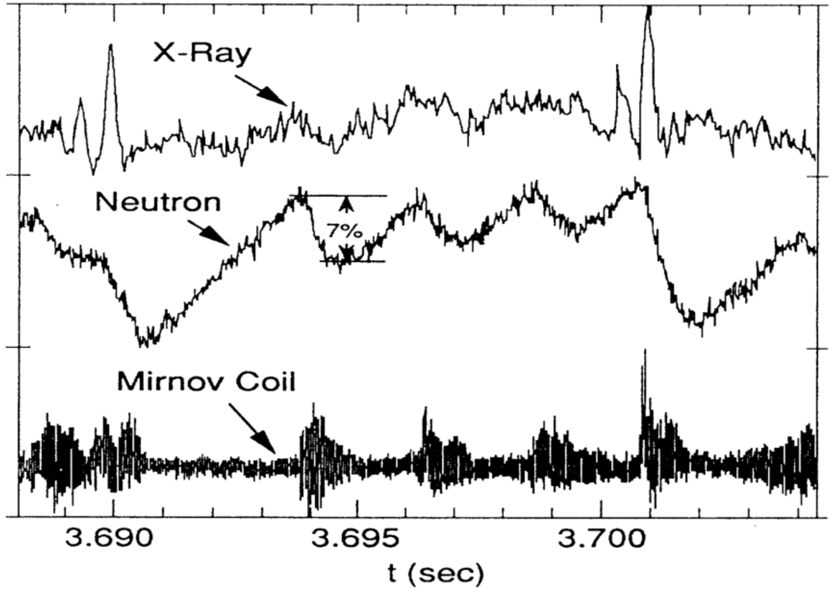
TAE: Toroidal Alfven Eigenmode

Wong et al PRL 1991



$$\omega = V_A / 2qR_0$$





TAE was observed by Mirnov coil and beam emission spectropy with NBI on TFTR.

BAE: Beta induced Alfven Eigenmode

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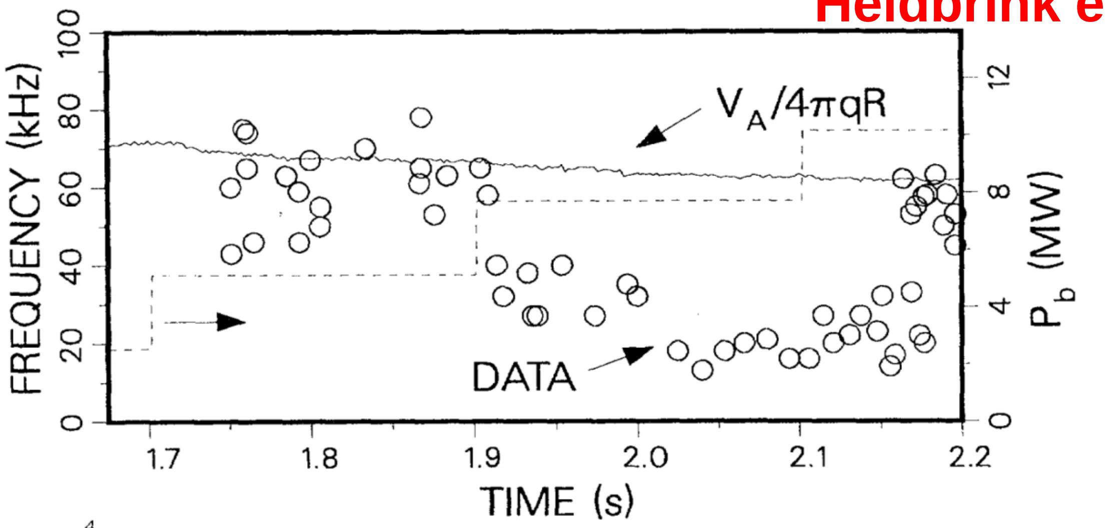
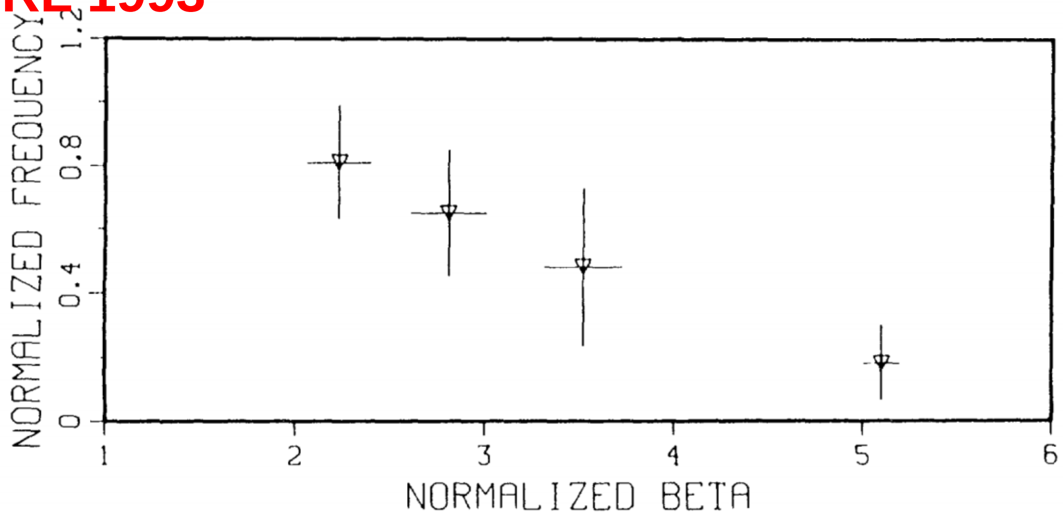
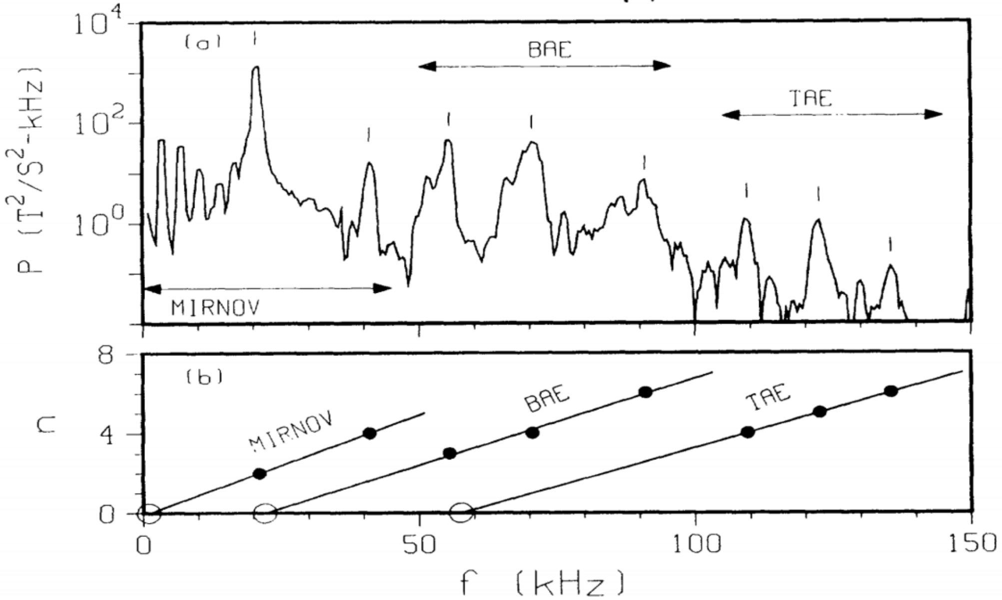
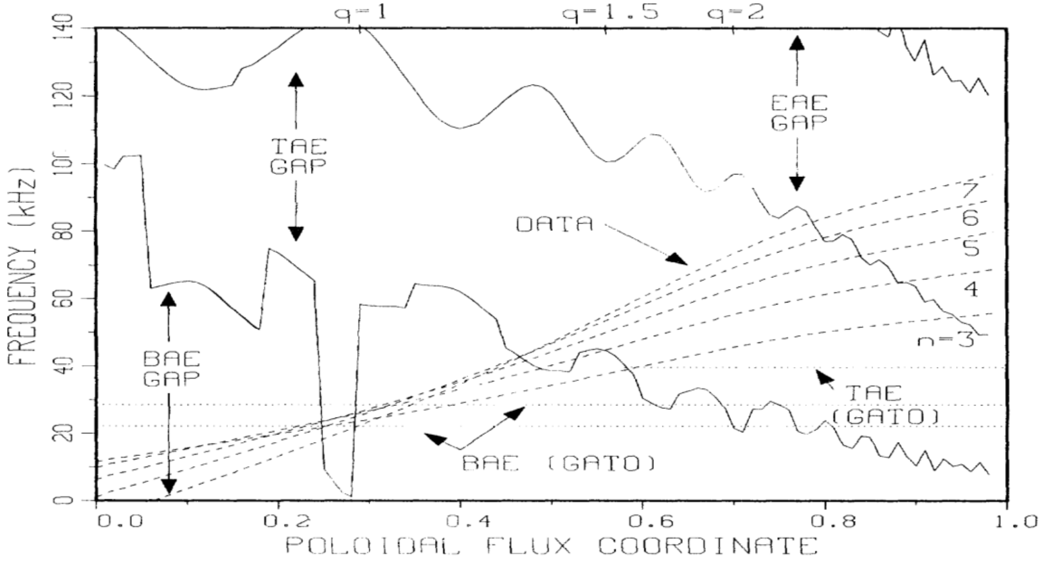
Heidbrink et al PRL 1993









BAE frequency is roughly half of TAE and decreases with β_N based on DIII-D experiments.

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Kinetic-MHD model in NIMROD

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$$\frac{\partial n_\alpha}{\partial t} + \nabla\left(n\boldsymbol{V}\right)_\alpha = \nabla \cdot \boldsymbol{D}\nabla n_\alpha$$

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + \boldsymbol{V}\cdot\nabla\boldsymbol{V}\right) = \boldsymbol{J}\times\boldsymbol{B} - \nabla p$$

$$+\nabla\cdot\rho\nu\nabla\boldsymbol{V} - \nabla\cdot\boldsymbol{\Pi} - \nabla p_h$$

$$\frac{n_\alpha}{\Gamma - 1}\left(\frac{\partial \boldsymbol{T}_\alpha}{\partial t} + \boldsymbol{V}_\alpha\cdot\nabla\boldsymbol{T}_\alpha\right) = -p_\alpha\nabla\cdot\boldsymbol{V}_\alpha$$

$$-\nabla\cdot\boldsymbol{q}_\alpha + \boldsymbol{Q}_\alpha - \boldsymbol{\Pi}_\alpha : \nabla\boldsymbol{V}$$

- **Resistive MHD**
- **Hall and 2-fluid**
- **Braginski and beyond closures**
- **Energetic particles**

$$\frac{\partial B}{\partial t} = \nabla\times\boldsymbol{E} + \kappa_{divb}\nabla\left(\nabla\cdot\boldsymbol{B}\right)$$

$$\boldsymbol{J} = \nabla\times\boldsymbol{B}$$

$$\boldsymbol{E} = -\boldsymbol{V}\times\boldsymbol{B} + \eta\boldsymbol{J}$$

$$+\frac{m_e}{n_e e^2}\left[\frac{m_e}{\mu_0}\left(\boldsymbol{J}\times\boldsymbol{B} - \nabla\boldsymbol{p}_e\right)\right.$$

$$\left.+\frac{\partial \boldsymbol{J}}{\partial t} + \nabla\left(\boldsymbol{J}\boldsymbol{V} + \boldsymbol{V}\boldsymbol{J}\right)\right]$$

Sovinec et al JCP 2004; Kim et al CPC 2004

Kinetic-MHD model in NIMROD

- **Momentum equation modified by hot particle pressure tensor:**

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{V}\right)=\boldsymbol{J}\times\boldsymbol{B}-\nabla P_b-\nabla\bullet\boldsymbol{P}_h+\nabla\cdot\rho\nu\nabla\mathbf{V}-\nabla\cdot\Pi$$

Energetic particle effect **Switch off**

- **Quasi-neutrality** $\Rightarrow n_e = n_i + n_h$

- **Steady state equation** $\boldsymbol{J}_0\times\boldsymbol{B}_0=\nabla p_0=\nabla p_{b0}+\nabla p_{h0}$
  - ∇p_{b0} **is scaled to accommodate hot particles**
  - **Scalar pressure requires isotropic velocity distribution**

- **Alternative J_h current coupling possible**

Kim et al POP 2008

Kink mode driven by slowing down EP

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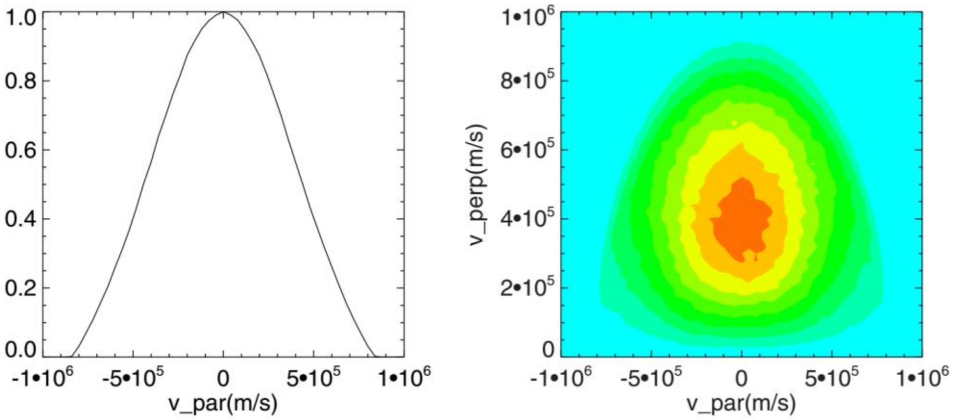
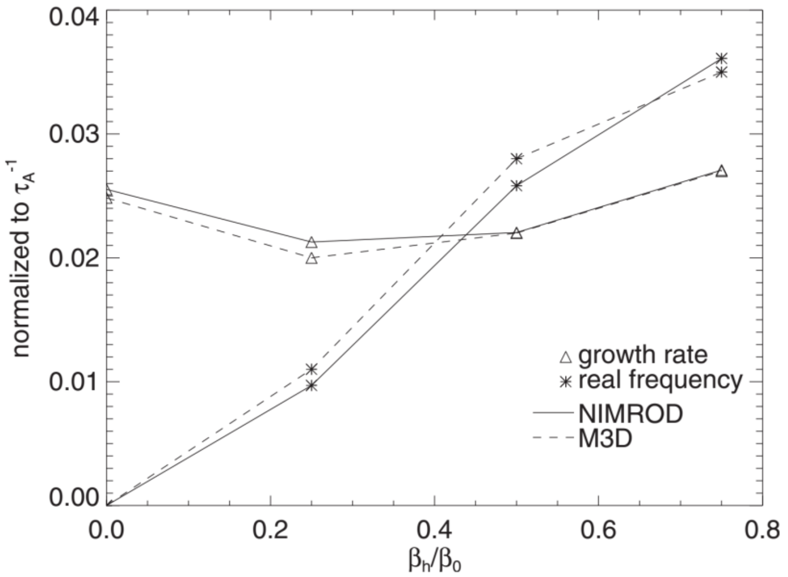
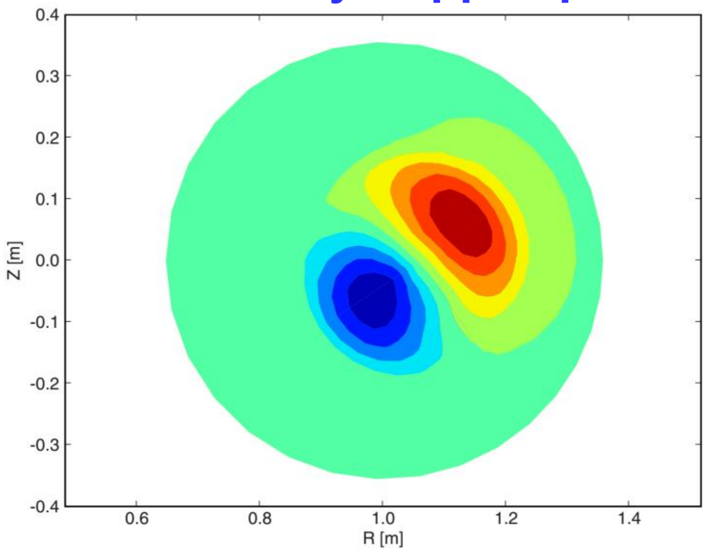
Initial slowing down EP distribution



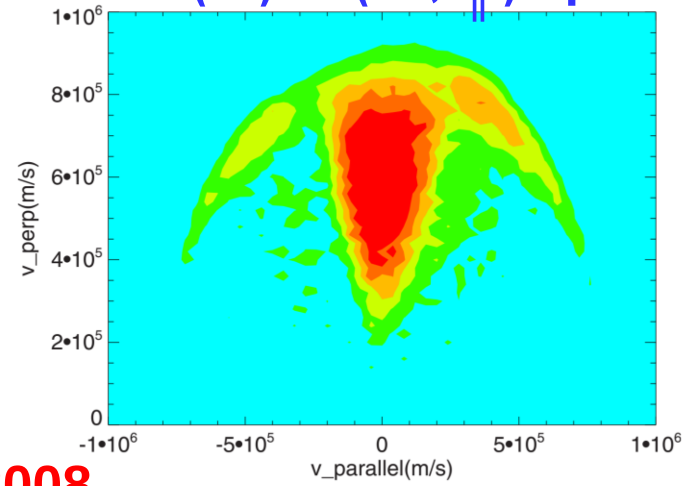
Benchmarked with M3D/M3D-k



n=1 anisotropic pressure contour mainly contributed by trapped particles

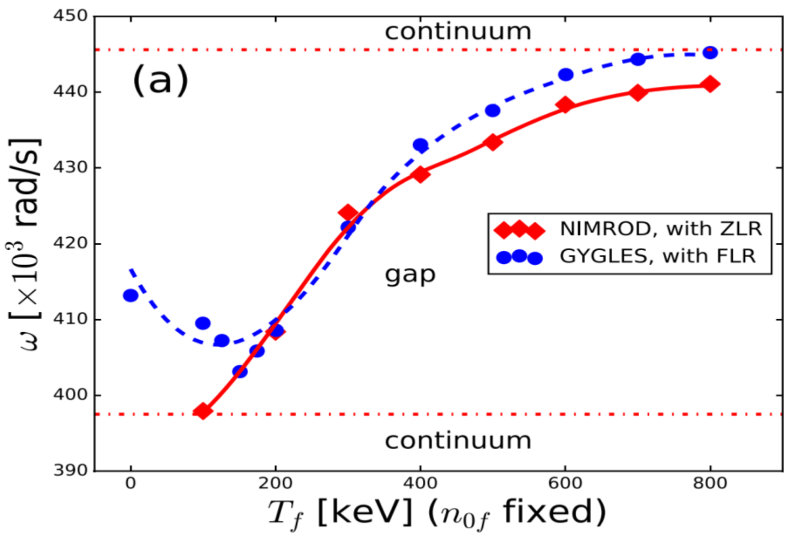


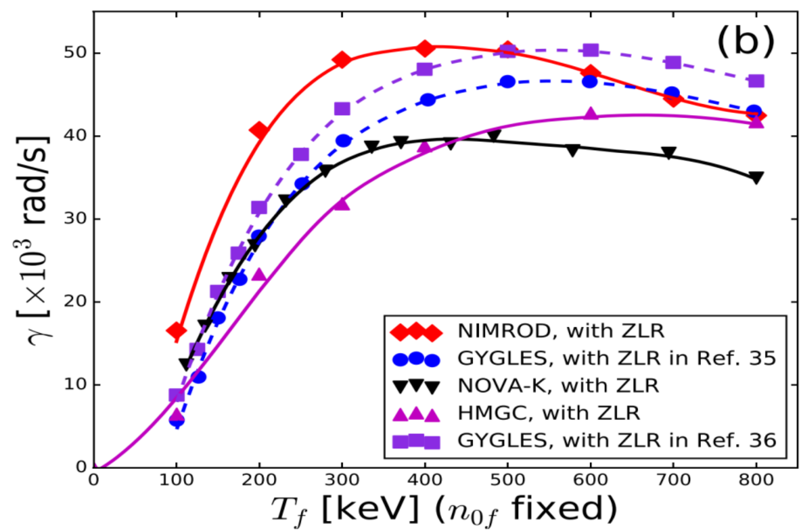
ABS(δf) in ($v_\perp$,$v_\parallel$) space



Kim et al POP 2008

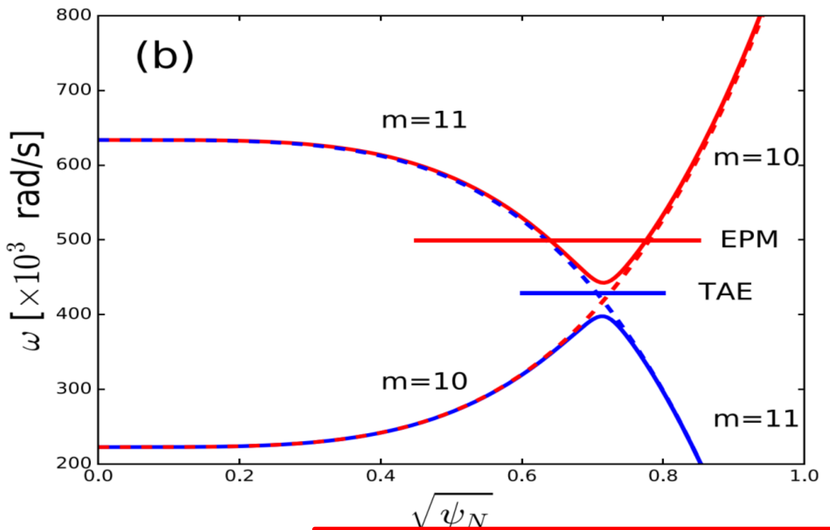
TAE driven by Maxwellian EP using NIMROD

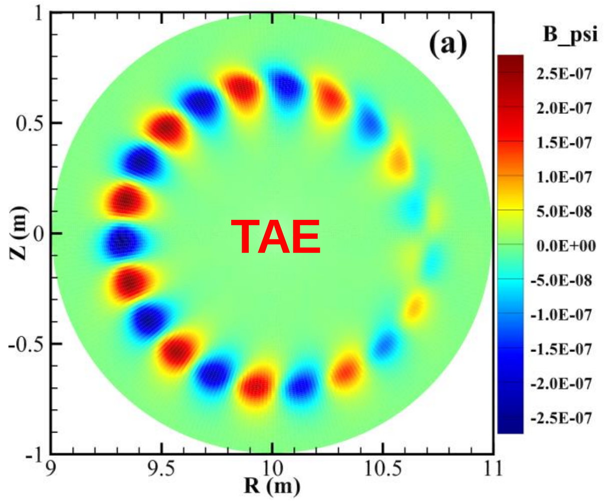


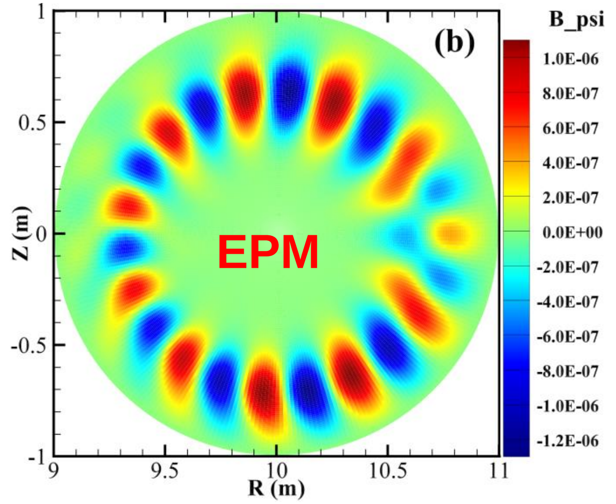


Hou et al POP 2018

Well benchmarked with NOVA-K, HMGC and GYGLES.







Transition from TAE to EPM occurs as EP β_f increases.

EAST shot #48916 & energetic particle model in NIMROD

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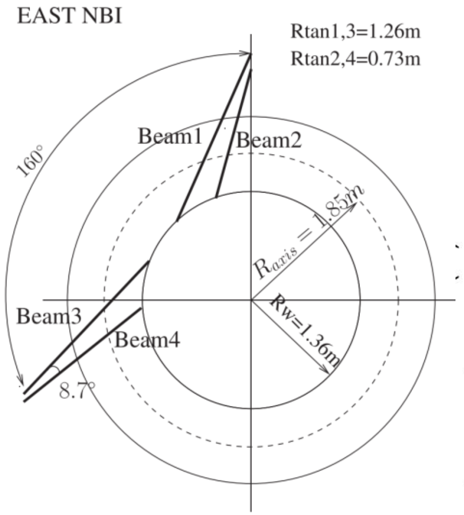
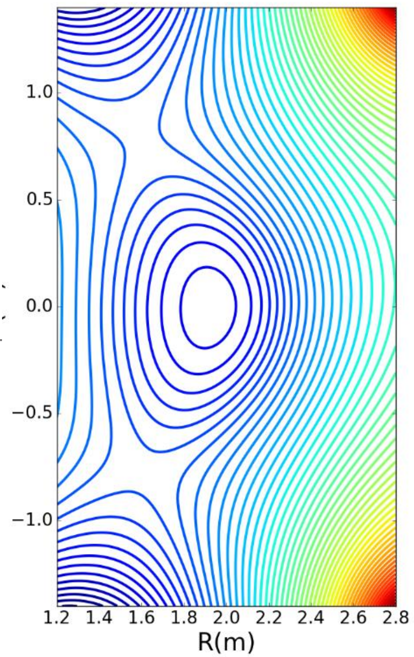
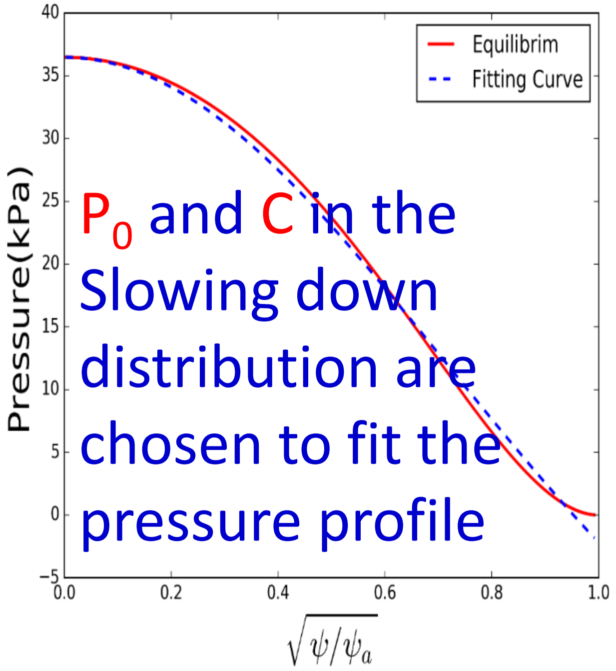
Hou et al POP 2019







P_0 and C in the Slowing down distribution are chosen to fit the pressure profile

- Energetic particles D+ is set to be slowing down distribution

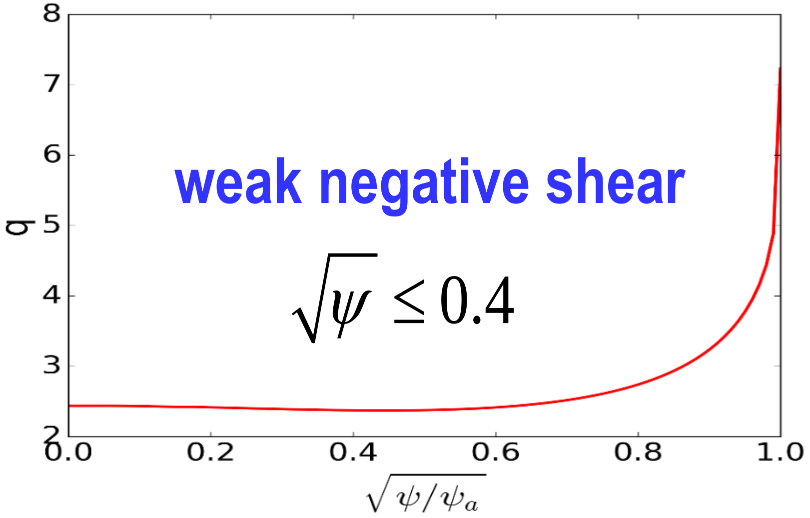
$$f_h = \frac{P_0 \exp\left(\dfrac{g\rho_{\|} - \psi_p}{c\psi_0}\right)}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}$$

$$g = RB_\phi \qquad \rho_{\|} = \frac{mv_{\|}}{qB}$$

- Critical energy $\varepsilon_c = 37.3keV$

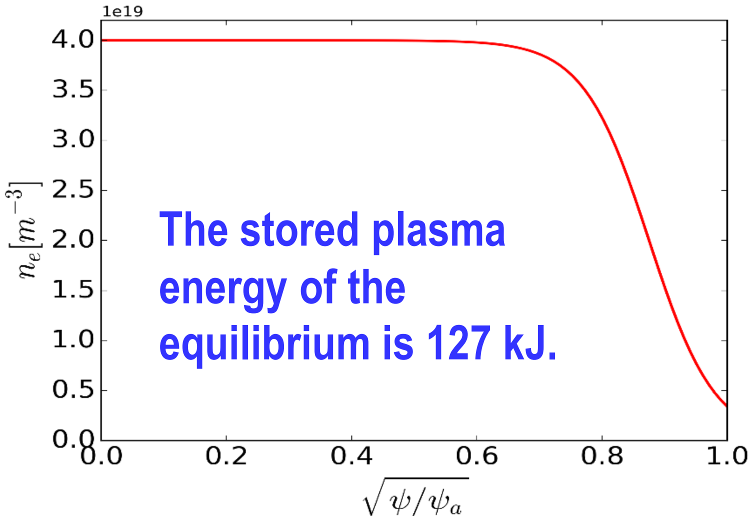
- Beta fraction $\beta_h/\beta = 0.2835$

EP stored energy: 36kJ



weak negative shear

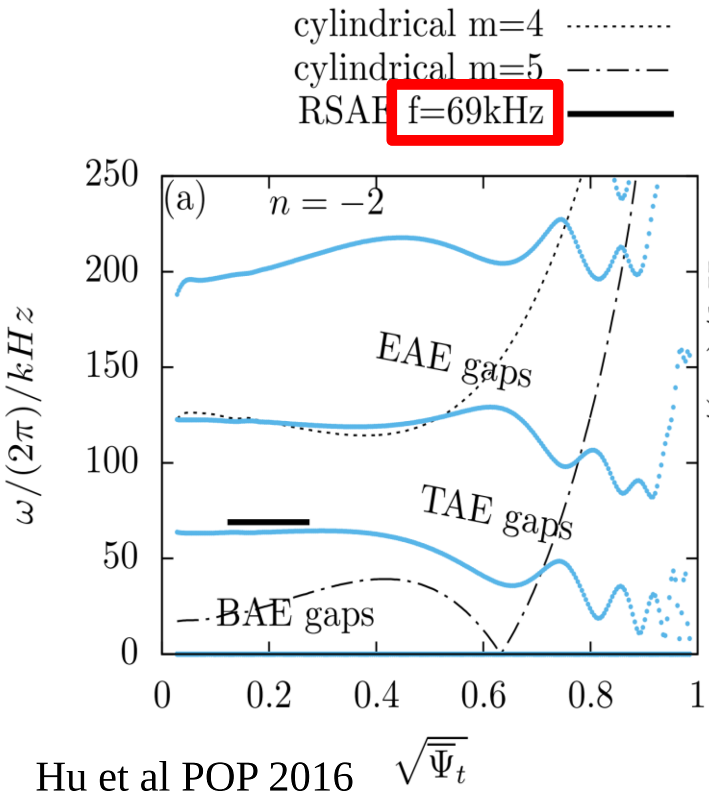
$\sqrt{\psi} \le 0.4$



The stored plasma energy of the equilibrium is 127 kJ.

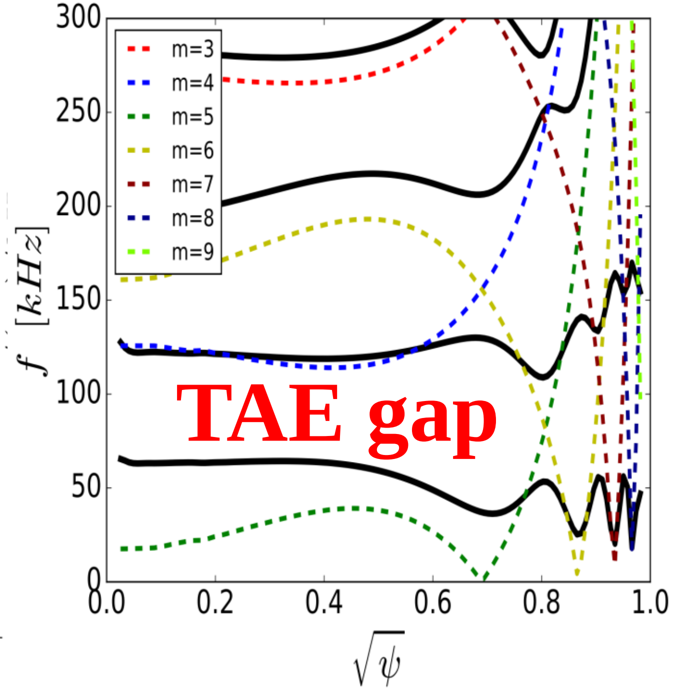
Continuum & mode structure - n=2

GTAW



Hu et al POP 2016

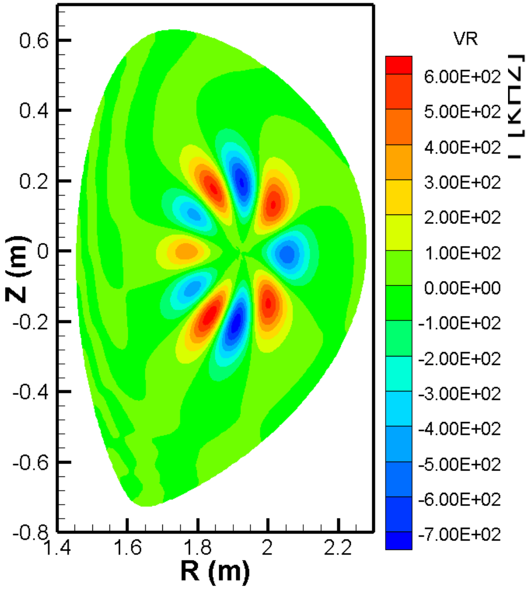
AWEAC



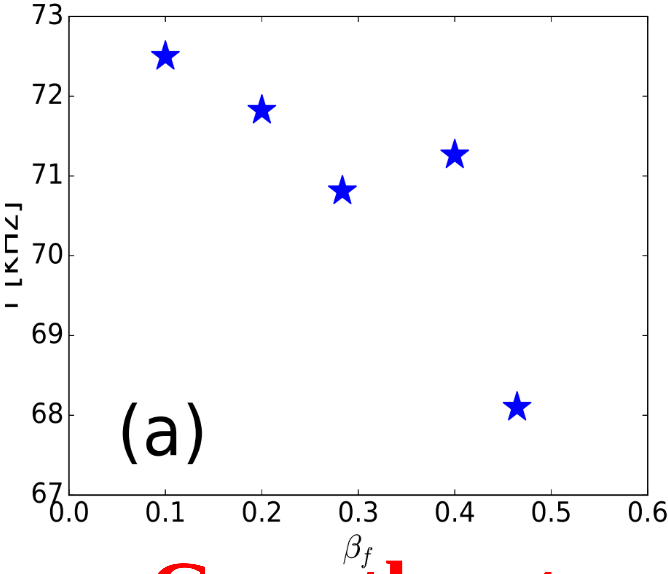
NIMROD

$\gamma = 2.395\times10^4 s^{-1}$,

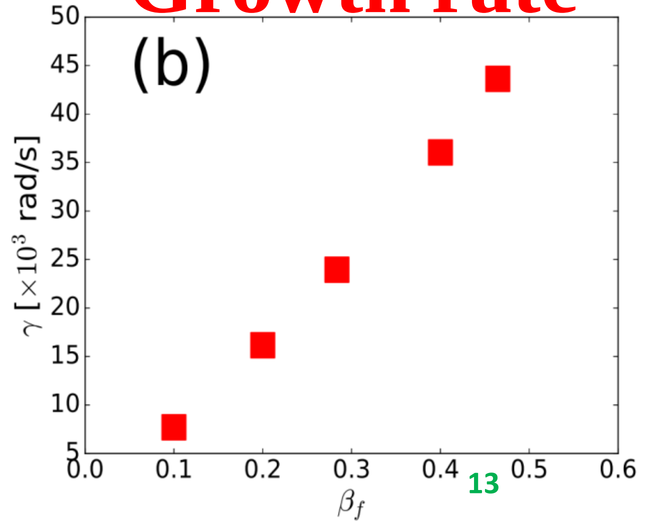
$f = 70.8\text{kHz}$



Frequency



Growth rate



TAE gap is induced by interaction of m=4 & 5 in radial range 0.2-0.4. RSAE with frequency around 70kHz is found from both eigen analysis and NIMROD simulation.

Contents

Main functions & progress of AWEAC

AWEAC: Alfven Wave Eigen-Analysis Code

□ Main functions:

- General tokamak geometry (asymmetric D shape); (achieved)
- Read equilibrium data (EFIT, TOQ, NIMEQ, etc.), output the characteristic parameters and visualizations; (achieved)
- Output the Alfven continuum (BAEs, TAEs, EAEs,…) with different mode number; (achieved)
- Achieve eigenmode growth rate and different damping rates, similar to NOVA/NOVA-k; (under development)
- Density effect due to fueling; flow effect; (under preparation)
- Platform for eigen-analysis in tokamaks. (future goal)

Ideal MHD eigenmode equations

- **Equilibrium eqs.**

$$\mathbf{J}\times\mathbf{B}=\nabla P$$
$$\nabla\times\mathbf{B}=\mathbf{J}$$
$$\nabla\cdot\mathbf{B}=\mathbf{0}$$

- **Linearized ideal MHD eqs.**

$$p_1+\boldsymbol{\xi}\cdot\nabla P+\gamma_s P\nabla\cdot\boldsymbol{\xi}=0$$
$$\rho\omega^2\boldsymbol{\xi}=\nabla p_1+\mathbf{b}\times\nabla\times\mathbf{B}+\mathbf{B}\times\nabla\times\mathbf{b}$$
$$\mathbf{b}=\nabla\times(\boldsymbol{\xi}\times\mathbf{B})$$

- **Ideal MHD eigenmode eqs.**

$$\nabla\psi\cdot\nabla\begin{pmatrix}P_1\\ \xi_\psi\end{pmatrix}=C\begin{pmatrix}P_1\\ \xi_\psi\end{pmatrix}+D\begin{pmatrix}\xi_s\\ \nabla\cdot\boldsymbol{\xi}\end{pmatrix}$$
$$E\begin{pmatrix}\xi_s\\ \nabla\cdot\boldsymbol{\xi}\end{pmatrix}=F\begin{pmatrix}P_1\\ \xi_\psi\end{pmatrix}$$

- **Where**

$$P_1=p_1+\mathbf{b}\cdot\mathbf{B}$$
$$\xi_s=\boldsymbol{\xi}\cdot(\mathbf{B}\times\nabla\psi)/|\nabla\psi|^2$$
$$\xi_\psi=\boldsymbol{\xi}\cdot\nabla\psi$$

Cheng et al POP 1986, Hu et al POP 2014

Alfven wave continuum equations

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- **Ideal MHD Alfven continuum eq.**

$$E\begin{pmatrix}\xi_s\\ \nabla\cdot\boldsymbol{\xi}\end{pmatrix}=\begin{pmatrix}E_{11} & E_{12}\\ E_{12} & E_{22}\end{pmatrix}\begin{pmatrix}\xi_s\\ \nabla\cdot\boldsymbol{\xi}\end{pmatrix}=0$$

$$E=E_a+\omega^2E_b$$

$$E_a\begin{pmatrix}\xi_s\\ \nabla\cdot\boldsymbol{\xi}\end{pmatrix}=-\omega^2E_b\begin{pmatrix}\xi_s\\ \nabla\cdot\boldsymbol{\xi}\end{pmatrix}$$

- **Where**

$$E^a_{11}=-\mu_0^{-1}\mathbf{B}_0\cdot\nabla\left(\frac{|\nabla\psi|^2\,\mathbf{B}_0\cdot\nabla}{B_0^2}\right)$$

$$E^a_{12}=-2\gamma P_0\kappa_s,$$

$$E^a_{21}=0,$$

$$E^a_{22}=\frac{\gamma P_0}{-\mu_0\rho_0}\mathbf{B}_0\cdot\nabla\left(\frac{\mathbf{B}_0\cdot\nabla}{B_0^2}\right)$$

$$E^b_{11}=-\frac{\rho_0|\nabla\psi|^2}{B_0^2}$$

$$E^b_{12}=0,$$

$$E^b_{21}=\kappa_s$$

$$E^b_{22}=\frac{\gamma P_0+\mu_0^{-1}B_0^2}{B_0^2}$$

$$\kappa=\mathbf{b}_0\cdot\nabla\mathbf{b}_0=(\nabla\times\mathbf{b}_0)\times\mathbf{b}_0$$

$$\kappa_s=2\boldsymbol{\kappa}\cdot\frac{\mathbf{B}_0\cdot\nabla\psi}{B_0^2}$$

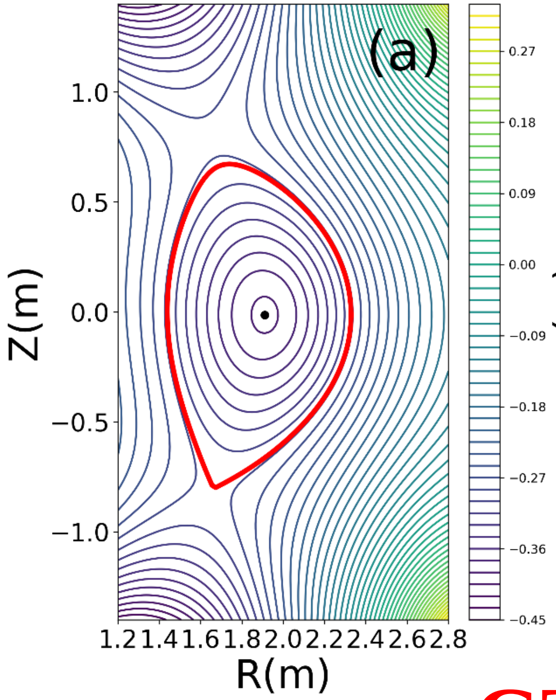
- **Slow sound approximation**

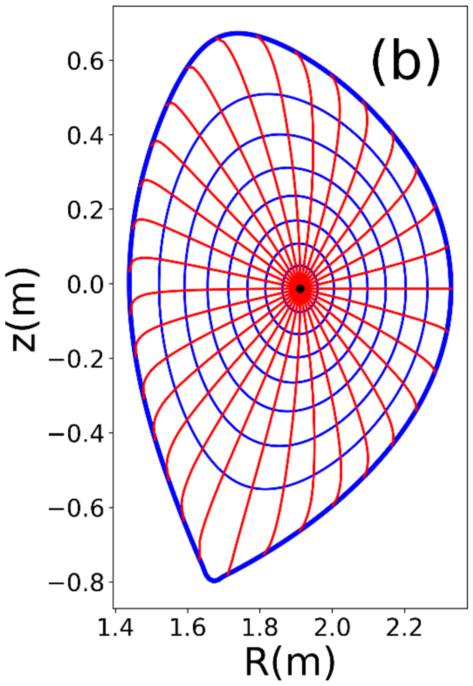
$$\frac{E^a_{22}}{E^b_{22}}=\frac{\gamma\beta/2}{1+\gamma\beta/2}$$

$$\beta\ll 1,\qquad E^a_{22}\to 0$$

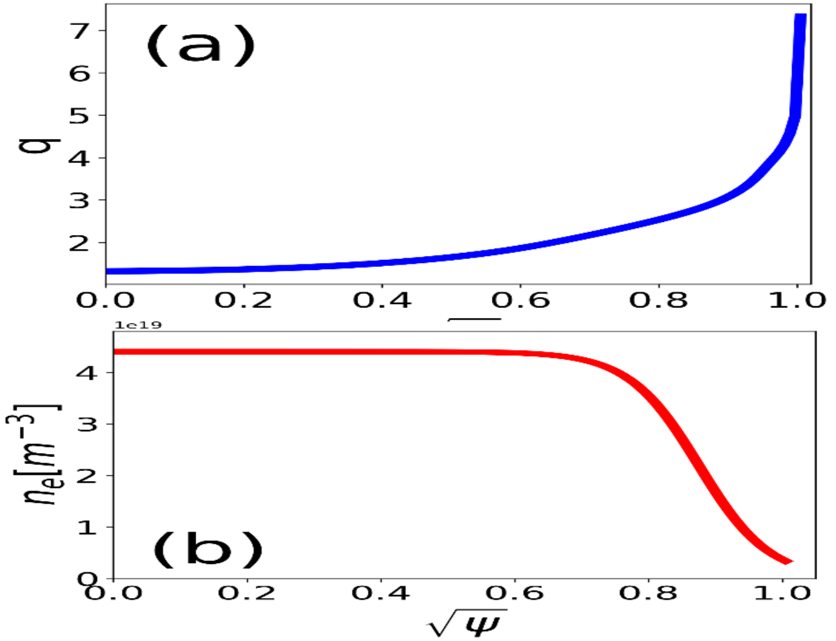
Hu et al POP 2014;
Deng et al NF 2012

Benchmark of AWEAC with GTAW



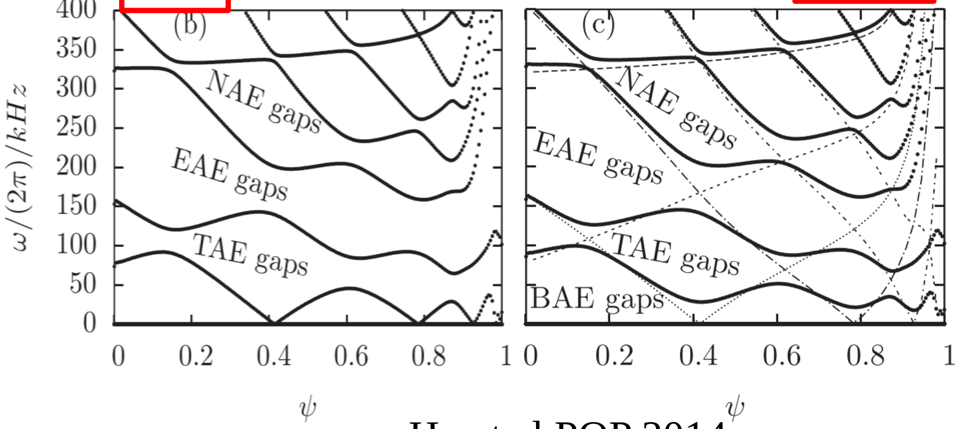


EAST shot #38300



GTAW

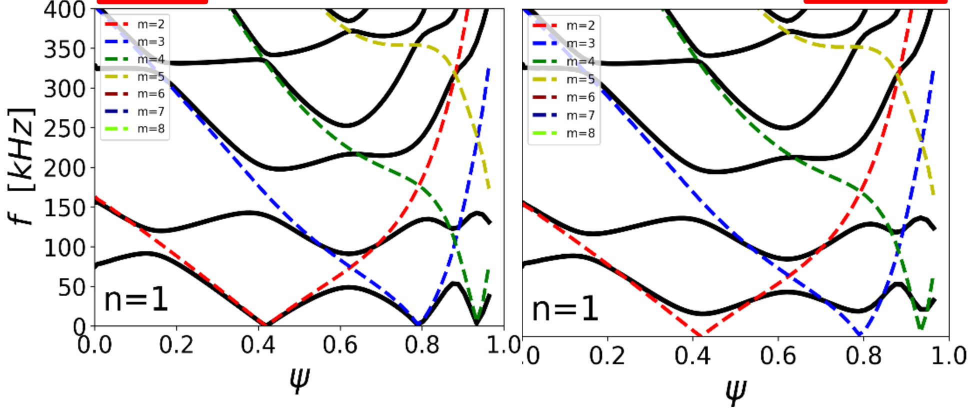
$\gamma=0$ $\gamma=5/3$



Hu et al POP 2014

AWEAC

$\gamma=0$ $\gamma=5/3$



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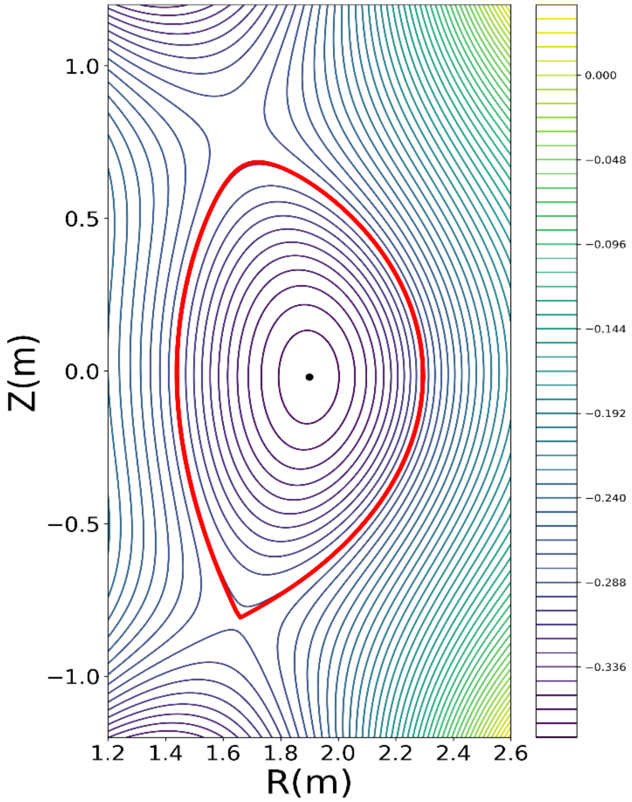
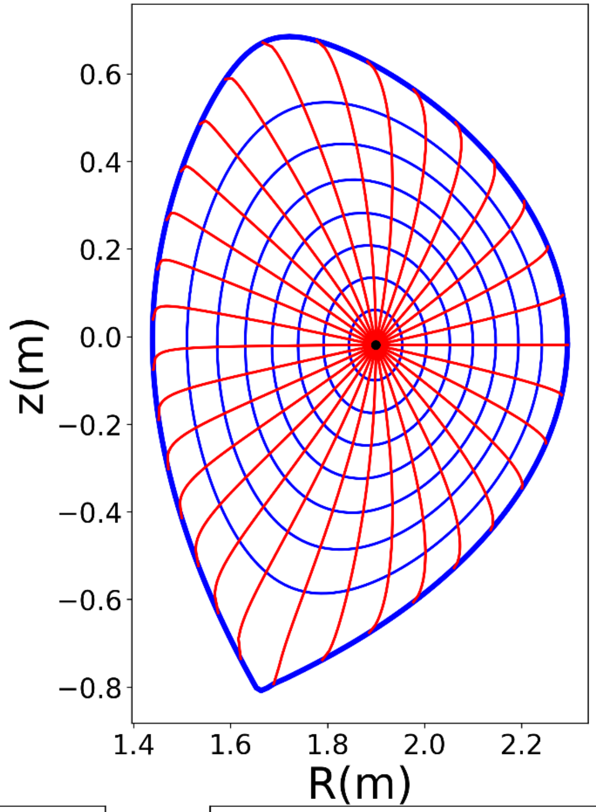
EP driven MHD modes with varied q_0





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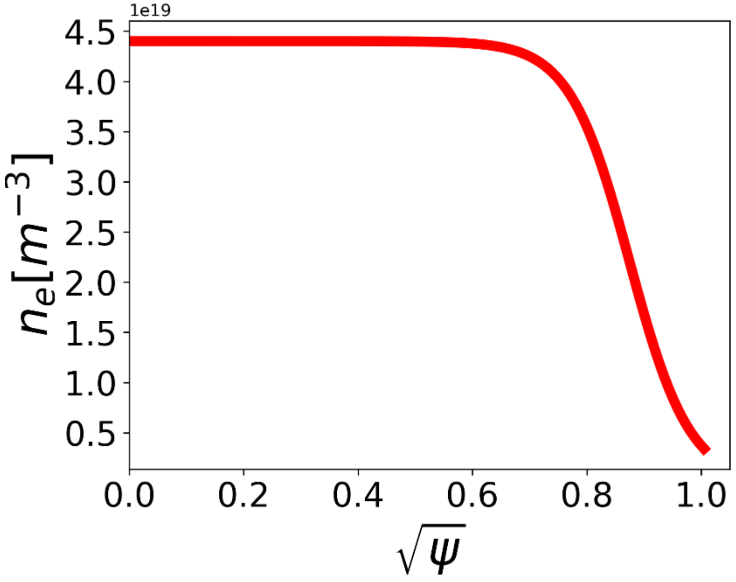
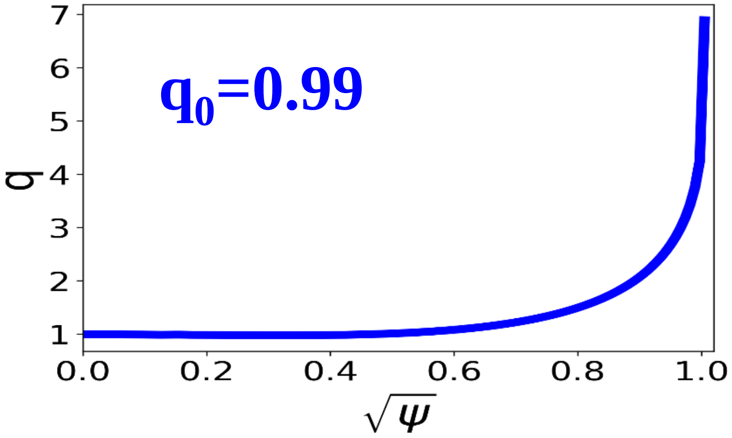
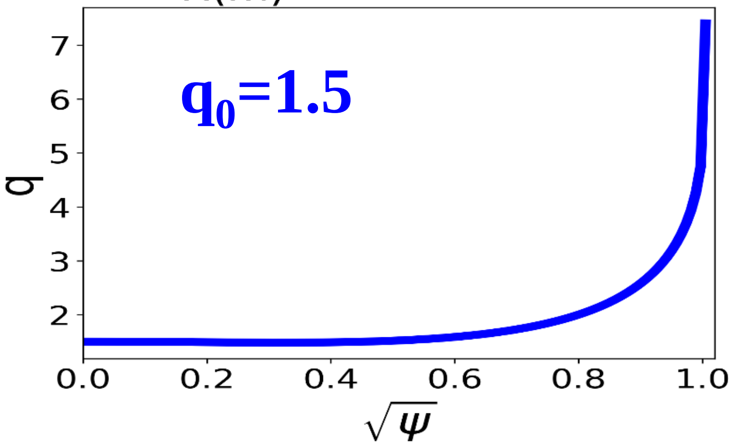
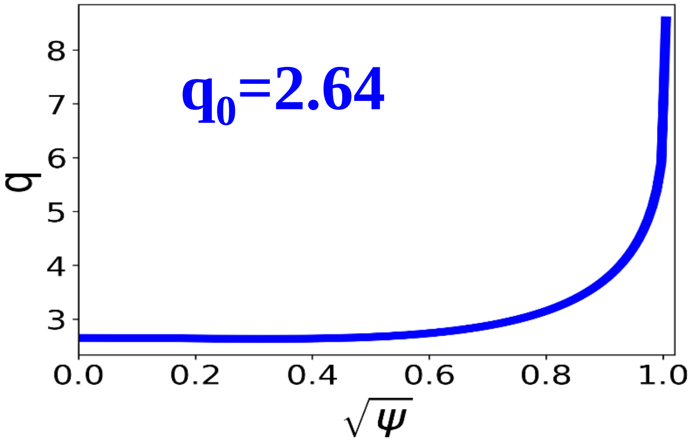
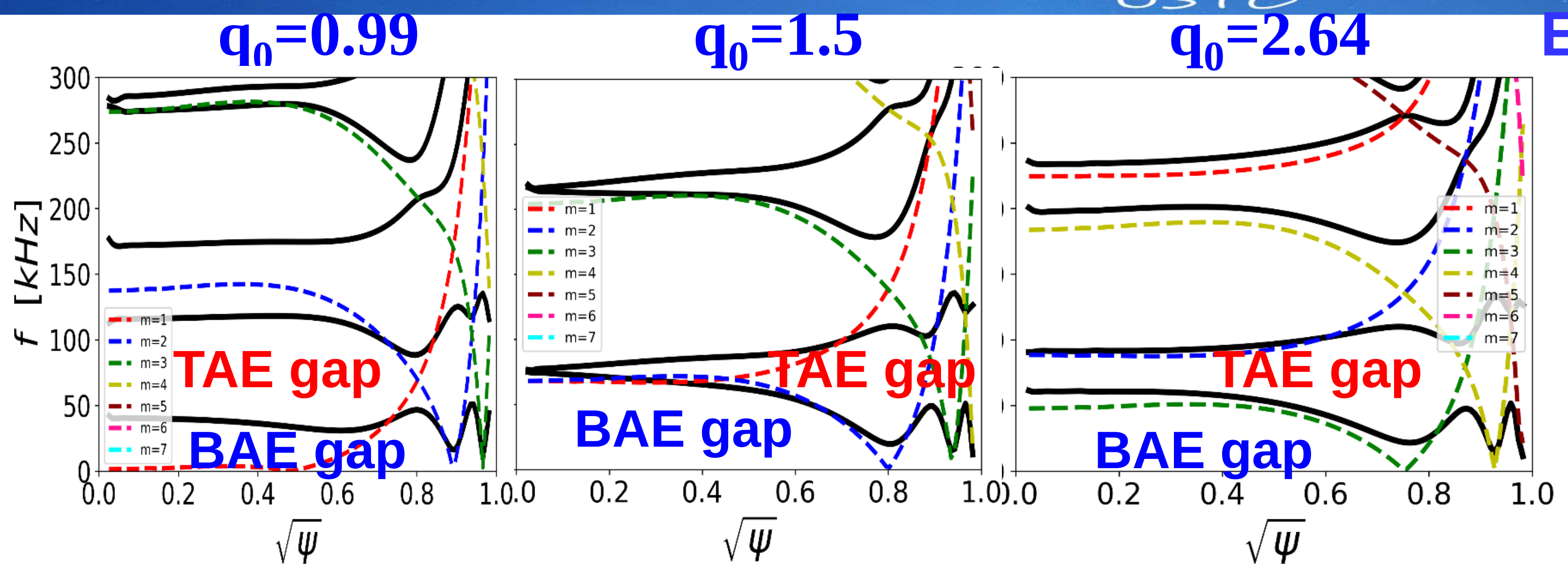








EP driven MHD modes with varied q_0



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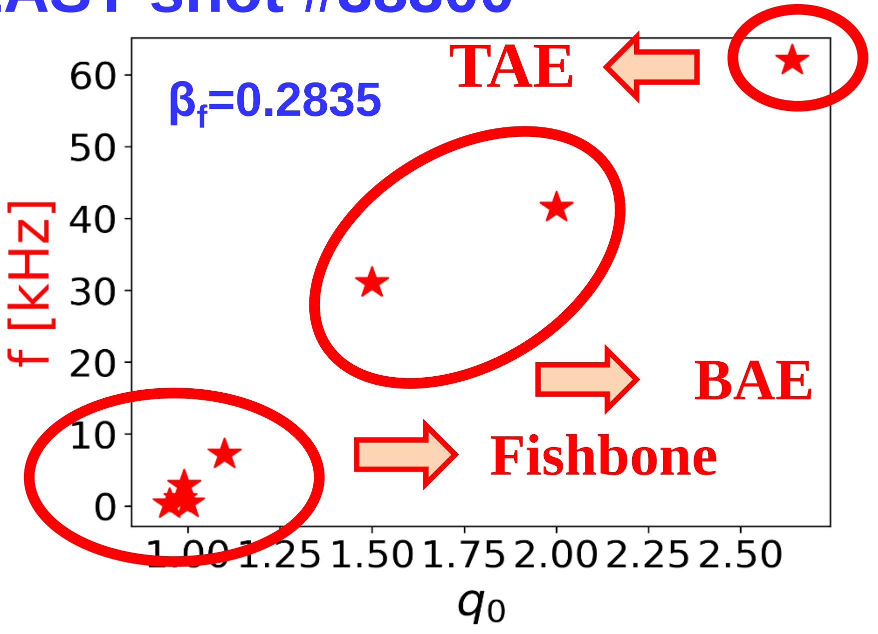
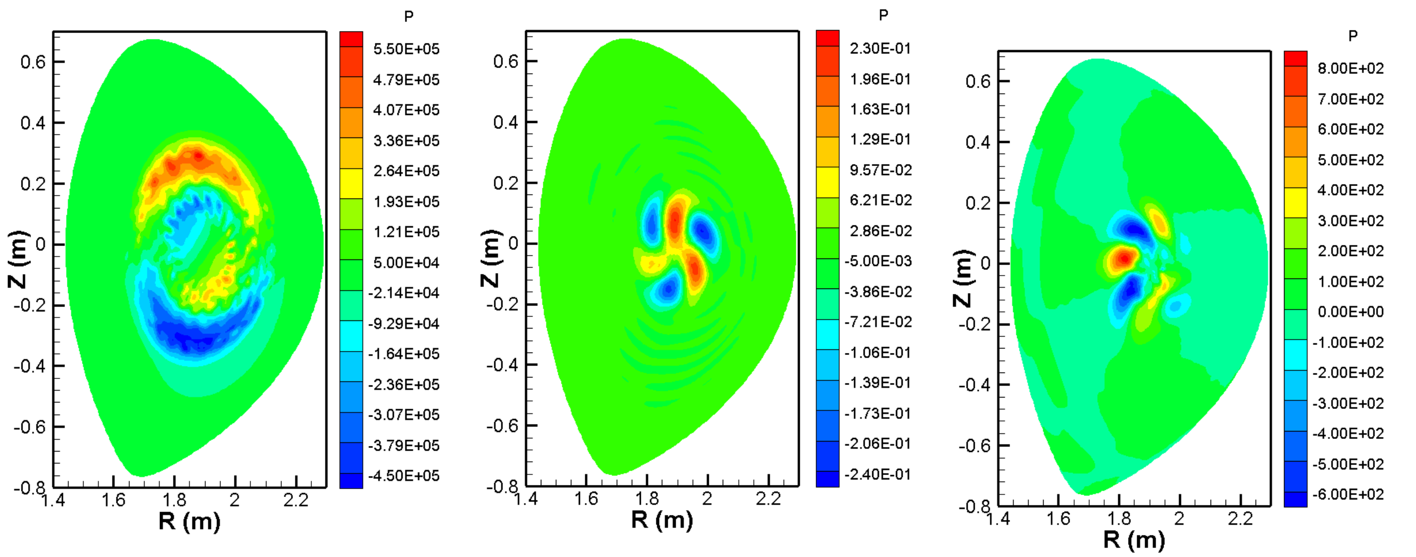
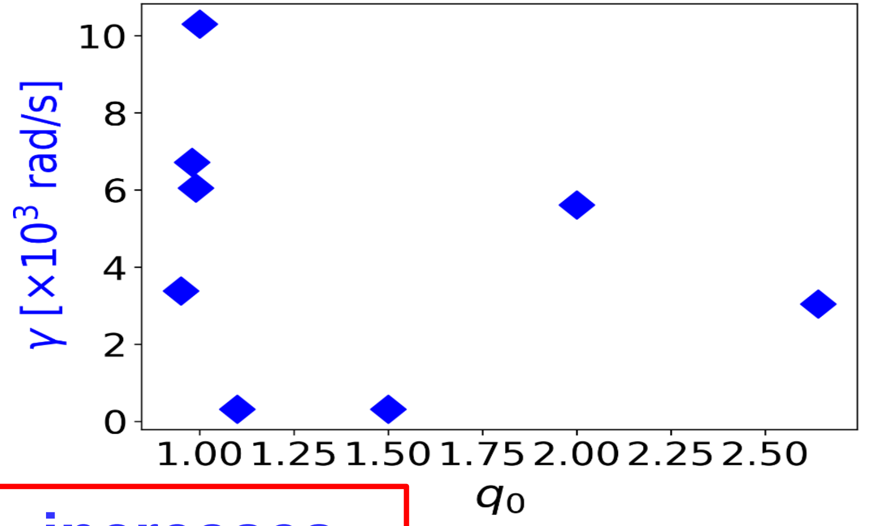
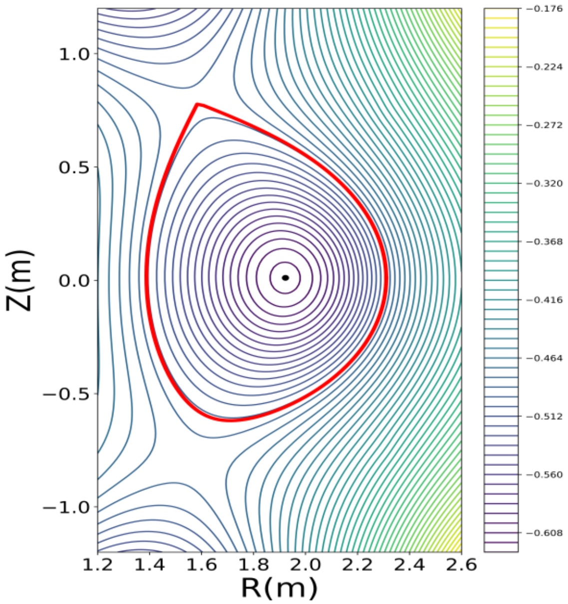


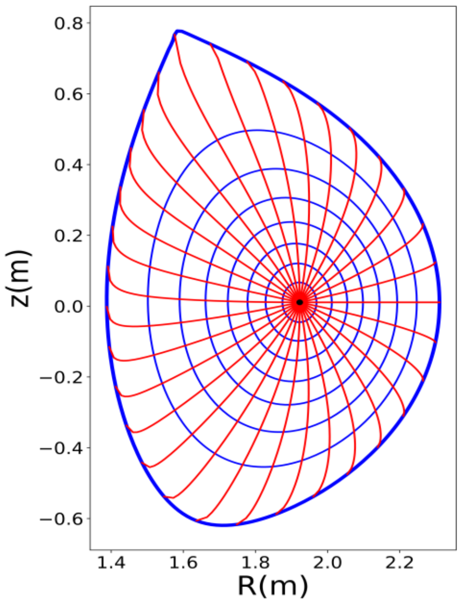




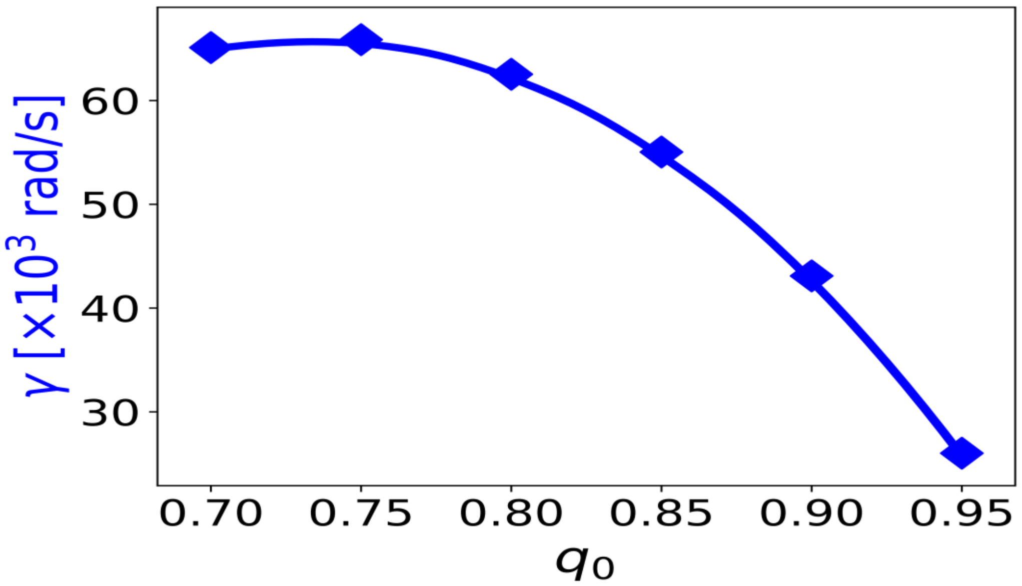
Transition from fishbone to BAE/TAE occurs as q_0 increases.

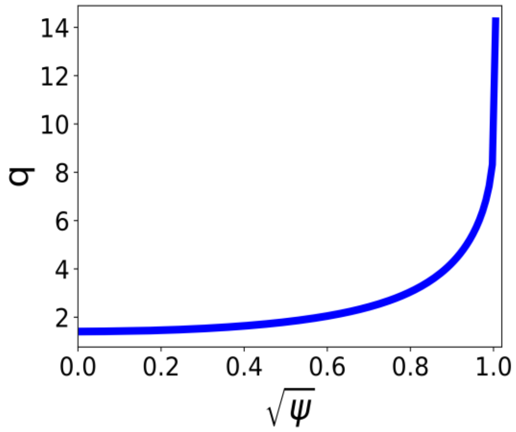
Kink/fishbone mode study of EAST shot #70187

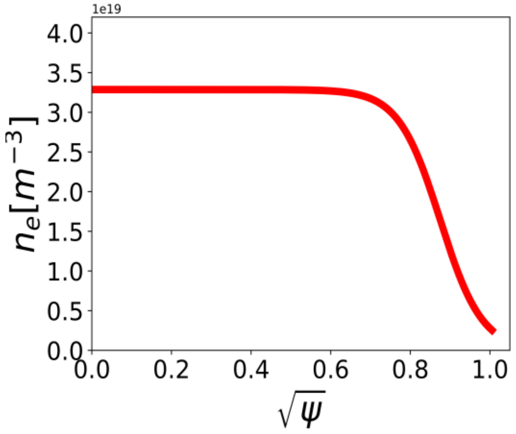


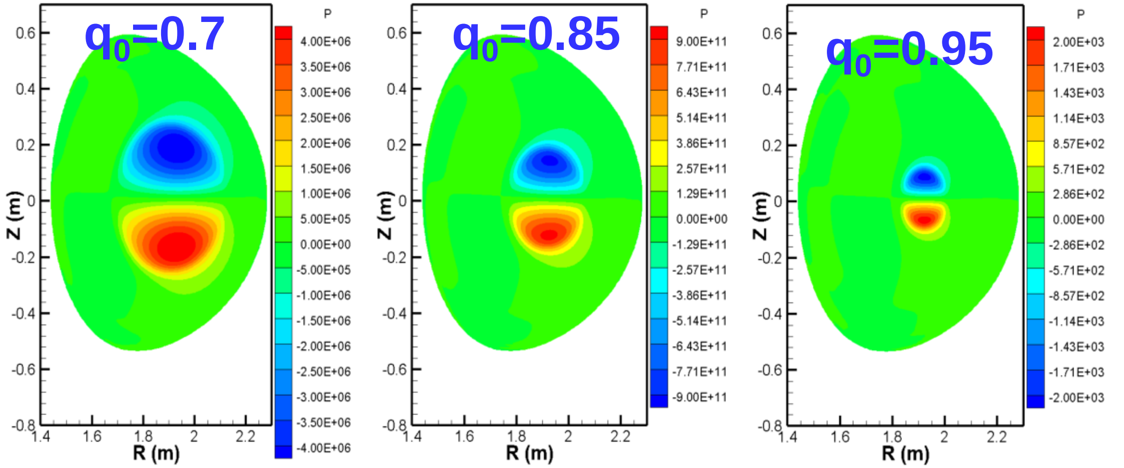


W/O EP



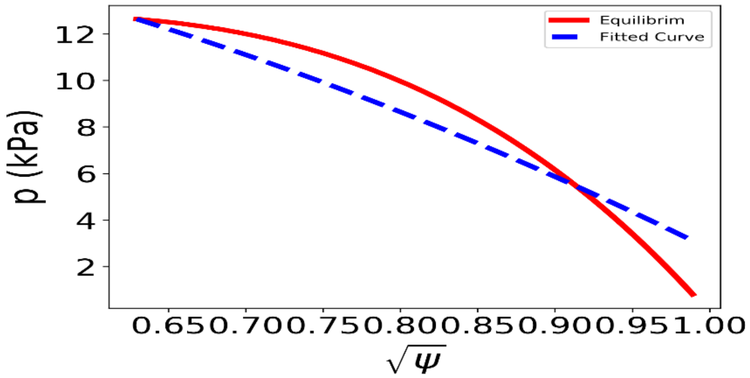






Kink mode increases first and then decreases as q_0 increases.

Kink/fishbone mode study of EAST shot #70187



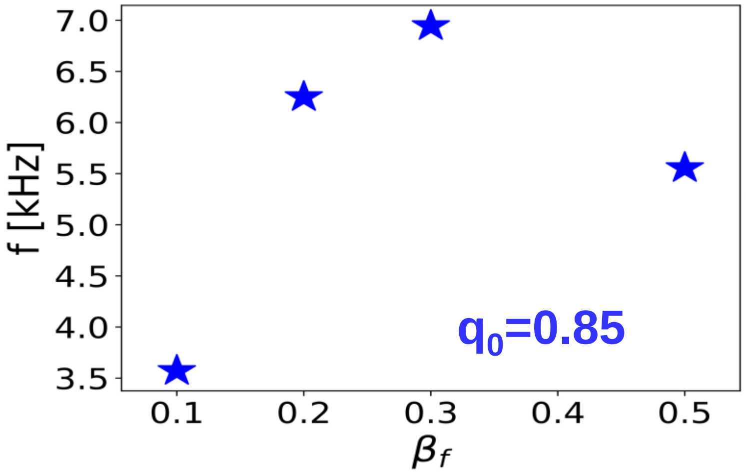
- Energetic particles D+ is set to be slowing down distribution

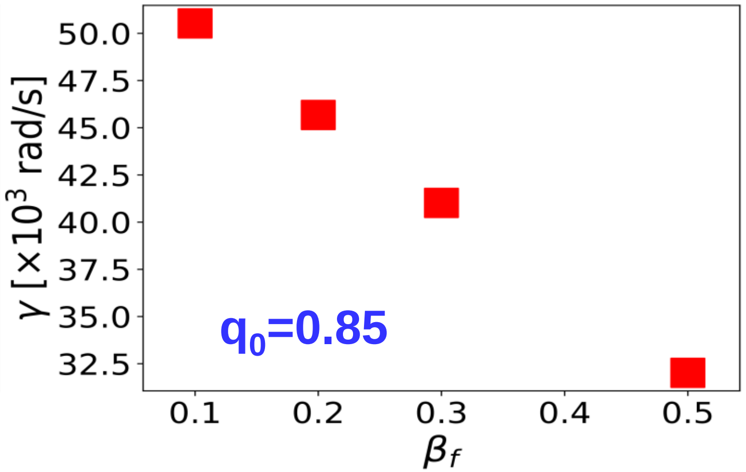
$$f_h = \frac{P_0 \exp\left(\frac{g\rho_{\|} - \psi_p}{c\psi_0}\right)}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}$$

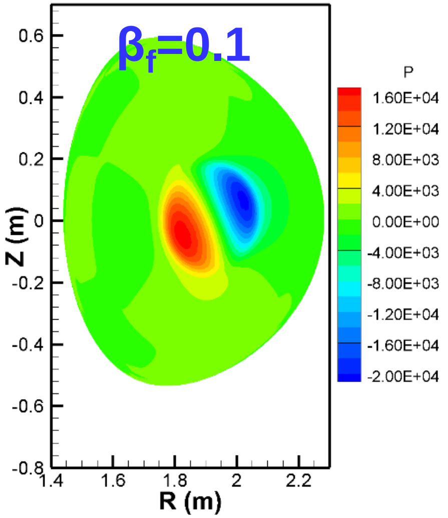
$$g = RB_\phi \qquad \rho_{\|} = \frac{mv_{\|}}{qB}$$

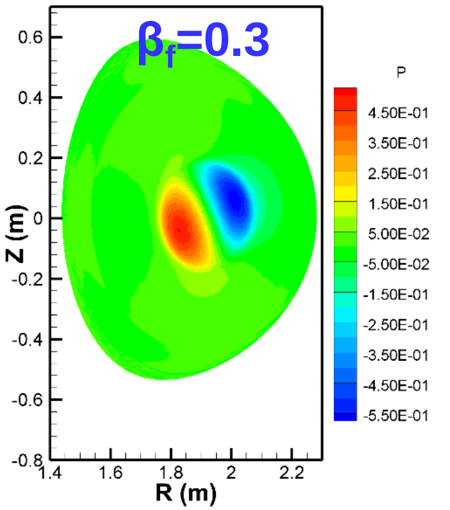
- Critical energy $\varepsilon_c = 37.3keV$

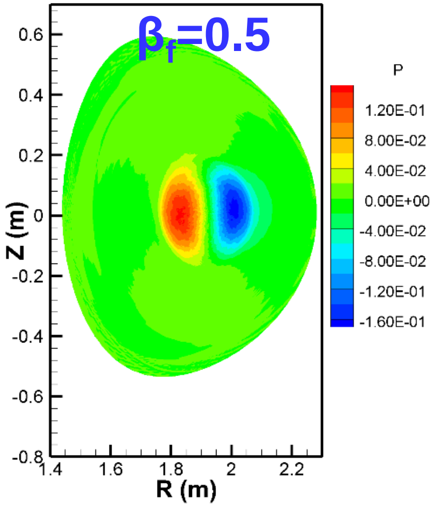
With EP











Fishbone mode is suppressed as β_f increases.

Contents

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- Motivation
- Kinetic-MHD model in NIMROD
- Functions & progress of AWEAC
- Fishbone/BAE/TAE on EAST
- Conclusions and future works

Conclusions & future works

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Conclusions:

- Kink/fishbone mode & TAE simulation using NIMROD has been benchmarked with both eigenvalue codes and initial value codes for both circular and EAST tokamaks.
- Transition from fishbone to BAE/TAE occurs as q_0 increases for EAST shot #38300;
- BAE is hard to be excited by EP for flat q profile with q_0 less than 1.

Future works:

- Simulations of fishbone and AEs on EAST/HL-2A based on experiment parameters, with energetic ions;
- Nonlinear simulations of EP driven AEs or MHD instabilities on EAST/HL-2A/ ITER/CFETR.

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Thank you!

Suggestions and collaborations are welcome!

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